

MU123

Discovering mathematics

HANDBOOK







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Handbook

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I Abbreviations

Some of the abbreviations used in MU123 are listed below. In the margin next to each entry is a reference to the unit and page of MU123 where the notation is first used.

AAA angle-angle condition for similar (not congruent) triangles

AAS angle-angle-side condition for congruent triangles

APR annual percentage rate

ASA angle-side-angle condition for congruent triangles

C8 31 ASS angle-side-side (not a condition for similar or congruent

triangles)

A1 13 BIDMAS brackets, indices, divisions and multiplications, additions and

subtractions

D14 199 CAST cosine, all, sine, tangent (mnemonic for which trigonometric

ratios are positive in which quadrants, starting from the bottom

right and going round anticlockwise)

A1 19 d.p. decimal place(s)

FOIL first, outer, inner and last (order for multiplying out brackets)

A3 124 HCF highest common factor

A4 202 IQR interquartile range

A3 121 LCM lowest common multiple

B5 35 LHS left-hand side of an equation

A4 180 PCAI pose question, collect relevant data, analyse the data, interpret

the results (the four stages of a statistical investigation)

B5 35 RHS right-hand side of an equation

C8 31 SAS side-angle-side condition for congruent triangles

A4 205 SD standard deviation

A1 20 s.f. significant figure(s)

A1 15 SI standard metric system (Système Internationale d'Unités)

D12 61 SOH CAH TOA sine is opposite over hypotenuse; cosine is adjacent over

hypotenuse; tangent is opposite over adjacent (mnemonic for

trigonometric ratios of side lengths in a right-angled triangle)

SSS side-side condition for congruent triangles

2 Notation

Some of the notation used in MU123 is listed below. In the margin next to each entry is a reference to the unit and page of MU123 where the notation is first used.

A1 20 \approx is approximately equal to

... ellipsis symbol, indicating that something has been omitted

A1 37 % percen

A2 98 and C8 9 ° a degree: an indication of a measurement on the Celsius or Fahrenheit temperature scales, or 1/360 th of a full turn.

A1 24

C8 31

<	less than	A2 103
\leq or \leqslant	less than or equal to	A2 104
>	greater than	A2 103
\geq or \geqslant	greater than or equal to	A2 104
a^n	a to the power n	A3 131
$0.1\dot{2}9\dot{6}$ or	$0.1\overline{296}$ the recurring decimal $0.1296296296296296\dots$	A3 137
a^{-n}	$1/a^n$, where $a \neq 0$	A3 144
士	plus or minus	A3 151
\sqrt{a}	the non-negative square root of a , where $a \geq 0$	A3 151
$\sqrt[n]{a}$	the non-negative <i>n</i> th root of a , where $a \geq 0$	A3 152
$a^{m/n}$	$(\sqrt[n]{a})^m$, where $a \ge 0$	A3 157
m:n	the ratio m to n	A3 159
Q1	the lower quartile	A4 202
Q3	the upper quartile	A4 202
(x, y)	the coordinates of a point (also used to label the point)	B6 63
P(x, y)	the point P with coordinates (x, y)	B6 63
x-, y-	prefixes placed before words like axis, coordinate and intercept. Here x is the standard variable associated with the horizontal axis, and y is the standard variable associated with the vertical axis, but other variables are often used in applications; the x or y in the prefix is then replaced by the appropriate variable.	B6 63
$y \propto x$	y is directly proportional to x	B6 88
r	the correlation coefficient	B6 107
\neq	not equal to	B7 127
AB	the line segment between points A and B , or its length	C8 9
$\angle ABC$	the angle formed from line segments AB and BC , or its size	C8 9
\widehat{ABC}	the angle formed from line segments AB and BC , or its size	C8 9
Ь	the square symbol indicating a right angle	C8 9
->>-	one or more arrowheads (on two or more lines) indicating parallel lines	C8 12
1	one or more arcs (on two or more angles) indicating equal angles	C8 14
$\triangle ABC$	the triangle with vertices A , B and C	C8 17
	one or more strokes (on two or more line segments) indicating line segments of equal length	C8 19
\cong	is congruent to	C8 30
π	the ratio of the circumference of a circle to its diameter (≈ 3.142)	A2 95 and C8 55
g	the acceleration due to gravity ($\approx 9.8\mathrm{m/s^2}$)	C10 129
$\sin \theta$	the sine of the angle θ	D12 61
$\cos \theta$	the cosine of the angle θ	D12 61
$\tan \theta$	the tangent of the angle θ	D12 61

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D12 69		$\sin^{-1}(x)$	or $\arcsin(x)$ the inverse sine of x
D12 69		$\cos^{-1}(x)$	or $arccos(x)$ the inverse cosine of x
D12 69		$\tan^{-1}(x)$	or $\arctan(x)$ the inverse tangent of x
D13 146		e	Euler's number (≈ 2.718)
D13 147		$\exp x$	the value e raised to the power x , that is, e^x
D13 152, 155		$\log x$	the logarithm of x to an unspecified base, or the logarithm to
			base 10 of x . (In some disciplines, $\log x$ can mean $\ln x$.)
D13 154		$\log_b x$	the logarithm to base b of x
D13 155		$\ln x$	the logarithm to base e of x
D14 224		x	the magnitude of x

3 Glossary

This glossary presents some of the key terms used in MU123. In the margin next to each entry is a reference to the unit and page of the book where the term is first used or defined. If appropriate this is followed by an italicised reference in brackets to a page in this handbook where further details can be found. Within definitions, cross-references to related glossary items are italicised.

alternate angles $\$ Two of the angles formed when a $line\ l$ crosses a

pair of parallel lines are said to be alternate if they lie between the parallel lines on opposite sides of l and have different vertices. Alternate angles on

A1 39	absolute comparison See relative comparison.
B6 74	absolute value See magnitude (of a number).
A2 78	accuracy (of an answer) How close the answer is to the correct value.
A4 213	accuracy (of a set of measurements) How close the average of a set of repeated measurements is to the true value.
C8 10	acute angle An angle greater than 0° and less than 90°.
D12 60	adjacent (angle and side) In a triangle, an angle and side are adjacent if the side is one of the two line segments that form the angle.
C8 23	adjacent (sides) Two sides of a shape that meet at a vertex of the shape.
B6 67 (35)	against On the graph of a formula, the dependent variable is usually put on the vertical axis and is said to be plotted against the independent variable, which is put on the horizontal axis.
Dr. 6	
B5 6	algebra A branch of mathematics in which letters are used to represent numbers.
B5 11 (34, 36)	algebraic expression See expression.
B5 30	algebraic fraction An algebraic <i>expression</i> that has the form of a <i>fraction</i> .
A2 76	algorithm A set of instructions to solve a problem step by step.

parallel lines are equal.

C8 14 (38)

amplitude (of a sinusoidal curve) Half the difference between the maximum and minimum values of the curve.	D14 226 (50)
angle There are three related meanings of the term angle: (1) a measure of rotation, often expressed in degrees or radians (see also sign of an angle); (2) a configuration consisting of two line segments emerging from a vertex; (3) the size of a rotation (usually the smallest) that makes one line segment of such a configuration lie in the same direction as the other.	C8 9 (38)
angle of elevation The angle between the horizontal and the line of sight to an object.	D12 59
angle of inclination The angle (greater than or equal to 0° and less than 180°) that a line makes with the positive x-axis, when the line is drawn on a pair of axes with equal scales.	D14 201 (50)
annual percentage rate (APR) The percentage of a loan, or of savings, that is to be paid as interest each year.	D13 134
antilogarithm The antilogarithm (to base b) of a number x is the number whose $logarithm$ (to base b) is x .	D13 149
apex (of a cone) See cone.	C8 59
apex angle The angle formed by the equal sides of an isosceles triangle.	C8 19
arc (of a circle) An unbroken section of the circumference of a circle.	C8 54
arccosine Another name for inverse cosine.	D12 69
arcsine Another name for inverse sine.	D12 69
arctangent Another name for inverse tangent.	D12 69
area (of a shape) The amount of surface that a shape occupies.	C8 48 (39)
arithmetic mean The arithmetic mean (or just mean) of a set of numbers is the sum of the numbers divided by however many numbers there are in the set.	A4 196 (33)
arithmetic progression Another name for arithmetic sequence.	C9 75 (40)
arithmetic sequence A sequence in which the difference between successive terms is constant. For example, 2, 2.5, 3, 3.5, 4,	C9 75 (40)
aspect ratio The aspect ratio of a rectangle is the ratio of its longer side to its shorter side.	A3 163
asymptote A <i>line</i> that a <i>graph</i> approaches but never reaches. An asymptote is often indicated on a graph by a dashed line.	D12 96
average speed The average <i>speed</i> is calculated by dividing the distance travelled by the time taken.	A2 72 (30)
axiom A truth that is taken as self-evident.	C8 8
axis of symmetry See line of symmetry.	C10 138 (43)
ballistics The science of projectiles.	C10 132
bar chart A diagram used to represent discrete numerical or categorical data. Each numerical or categorical item is represented by a rectangle, called a bar or column, whose length is proportional to the numerical value associated with that item (this could be its frequency of occurrence, or something else). The bars are of equal thickness and they have bases along either a horizontal or a vertical axis. There are equal gaps between the bars.	D11 22
base (number) See index form.	A3 131 (32)

base (of a shape) The side (or face) of a shape at right angles to C8 49 (39) which the *perpendicular height* is to be measured. base angles The two equal angles of an isosceles triangle. C8 19 base units Units of measurement from which all others are derived. A1 15best fit line See regression line. B6 105 An acronym that acts as a reminder of the order in which to BIDMAS A1 13 (28) carry out mathematical operations. binary data Data that can take only two values, often 1 and 0, and are A4 185 widely used to represent categories such as yes/no, pass/fail or true/false. To cut into two equal parts. C8 36 **boxplot** A diagram used to represent five key summary values of a D11 10 (45) dataset, namely minimum, lower quartile, median, upper quartile and maximum. A box is drawn between the lower and upper quartile to indicate the *interquartile range*, while two line segments (known as whiskers) are drawn between the box and the minimum and maximum values of the dataset to indicate its full range. cancelling (an algebraic fraction) The process of cancelling any C9 103 common factors of the numerator and denominator. cancelling (a numerical fraction) The process of dividing the top **A1 34** (29) and bottom of a fraction by a whole number (larger than 1) to obtain an equivalent fraction with a smaller numerator and denominator. cancelling out terms Two or more like terms of an expression cancel B5 18 (35) each other out if their coefficients add up to zero. **capacity** The amount of liquid that an object can contain. C8 60 Cartesian coordinate system A way of specifying the position of a B6 63 point using coordinates. categorical data Data that have been classified according to a set of D11 22 categories. For example, the following categories might be used in connection with housing: Detached houses, Semi-detached houses, Terraced houses, Purpose-built flats, Converted flats and Other. centre (of a circle or sphere) See circle and sphere. C8 54 centre (of a circular arc) The centre of the circle on whose D12 104 circumference the arc lies. C8 27 centre of rotation The *point* about which something is rotated. **chance** Another name for *probability*. D14 181 chord A line segment starting and ending on the circumference of a C8 54 circle. circle A plane shape whose boundary consists of all points that are a C8 54 (39) fixed distance from a fixed point called the centre of the circle. The word circle is also used to refer to the boundary of such a shape. circular arc Another name for arc (of a circle). D12 104 circumference (of a circle) The boundary of a circle, or the length C8 54 (39) of the boundary. circumscribe To construct (a shape) around another shape so that it C8 56 touches but does not cross that other shape.

clearing a fraction The process of removing a <i>fraction</i> (numerical or algebraic) from an <i>equation</i> by multiplying both sides of the equation by a suitable number or expression. (The number must be non-zero, and the	B5 43 and C9 110 (35, 4	2)
expression must be non-zero for all values of the variables under consideration.)		
coefficient When a <i>term</i> in an <i>expression</i> consists of a number (including any signs) multiplied by a combination of letters, the number is called the coefficient of the term.	B5 15	
coefficient (of a quadratic expression) The coefficients of the quadratic expression $ax^2 + bx + c$ are the constants a , b and c .	C9 87	
collecting like terms The process of combining like terms of an expression into a single term.	B5 16 (35)	
common denominator A common denominator of two or more fractions is a common multiple of their denominators.	A3 138 and C9 105 (31,	42)
common difference The difference between successive terms in an arithmetic sequence.	C9 75 (40)	
common factor (of integers) A common factor of two or more <i>integers</i> is an integer that is a <i>factor</i> of them all.	A3 124 (31)	
common factor (of terms) A common factor of two or more <i>terms</i> is an <i>expression</i> that is a <i>factor</i> of them all.	B7 134 (36)	
common logarithm The common logarithm of a number x is the power to which 10 has to be raised to obtain x (that is, the logarithm of x to base 10). This is sometimes written with the subscript omitted, as $\log x$.	D13 150	
common multiple (of expressions) A common multiple of two or more <i>expressions</i> is an expression that is a <i>multiple</i> of them all.		
common multiple (of integers) A common multiple of two or more <i>integers</i> is an integer that is a <i>multiple</i> of them all.	A3 121 (31)	
common side A <i>line segment</i> that is a side of two shapes.	C8 36	
comparative bar chart A <i>bar chart</i> in which there are two or more bars for each data item. For example, two bars could be associated with each year, one to represent the number of mobile phones, and the other the number of land lines.	D14 186	
completed-square form Every quadratic expression $ax^2 + bx + c$ can be rearranged into the form $a(x + a \text{ number})^2 + a \text{ number}$. This is known as the completed-square form of the quadratic expression.	C10 162 (44)	
completing the square The process of rearranging a quadratic expression into its completed-square form.	C10 162 (44)	
composite number An integer greater than 1 that is not a prime.	A3 126	
compound interest Interest that is a <i>percentage</i> of both the initial amount of an investment and all the interest accumulated so far.	D13 122	
compound unit A unit of measurement that involves more than one of the <i>base units</i> , such as m/s or m ³ .	A2 72	
cone A three-dimensional shape with a circular base, whose cross-sections (parallel to the base) are circles that decrease in radius uniformly to a point, known as the apex of the cone. The centres of the circular cross-sections form a straight line perpendicular to the base.	C8 59 (40)	
congruent Geometric <i>figures</i> with the same size and shape (possibly flipped) are said to be congruent.	C8 28 (39)	
		0

A1 46 **conjecture** An informed guess about what might be true, often obtained by considering some special cases. B6 88 A constant in an equation or expression is a quantity that does not change when the values of the variables change. Sometimes 'constant' is used as a short form of constant term. constant of proportionality See direct proportion. B6 88 **constant term** A term, in an expression, that is just a number. B5 15 **construction** An addition to a geometric *figure*, used to help prove a C8 17 result about the original figure. **construction line** A line used as (part of) a construction. C8 17 continuous data Data that can take all the 'in-between' values on a A4 186 number scale. continuous exponential change See exponential change. D13 125, 137 **converse** The reverse of a mathematical statement. The converse of the C8 16 statement 'If A is true, then B is true' is the statement 'If B is true, then A is true. **conversion graph** A graph used to convert from one unit to another A2 84 unit, for instance m/s to km/h. coordinates A pair of numbers used to represent a point. The first A2 85 and B6 63 number specifies the position of the point along the horizontal axis from 0, and the second number specifies its position along the vertical axis from 0. These numbers are known as the horizontal coordinate and the vertical coordinate, respectively. **correlation** See positive correlation, negative correlation, B6 108 perfect correlation. correlation coefficient The correlation coefficient of a set of paired B6 107 data measures how closely the regression line fits the data points. A value close to +1 indicates a strong positive correlation, whereas a value close to -1 indicates a strong negative correlation; the closer the value is to 0, the weaker is the correlation. A correlation coefficient is sometimes denoted by r. corresponding angles (on parallel lines) Two of the angles C8 14 (38) formed when a line l crosses a pair of parallel lines are said to be corresponding if they have different vertices and lie on the same side of lwith just one of them between the parallel lines. Corresponding angles on parallel lines are equal. corresponding angles or vertices (of congruent triangles) C8 30 (39) An angle (or vertex) in one triangle is said to correspond to an angle (or vertex) in a congruent triangle if the two angles (or two vertices) can be made to coincide by superimposing the two triangles (flipping one if necessary). corresponding sides (of congruent or similar triangles) C8 34 If two triangles have the same three angles, then a side in one triangle is said to correspond to a side in the other triangle if they are opposite equal angles. D12 61, 90 (45) The cosine of an angle θ , written $\cos \theta$, is the x-coordinate of the

point obtained by rotating the point (1,0) about the origin through the angle θ . For an acute angle θ in a right-angled triangle, $\cos \theta$ is equal to the length of the side adjacent to θ divided by the length of the hypotenuse.

cosine curve The graph of the cosine function.	D12 95
Cosine Rule A rule for finding the length of one side of a <i>triangle</i> given the lengths of the other two sides and an <i>angle</i> , or for finding an angle given the lengths of the three sides.	D12 80 (45)
critical region In certain types of statistical test, the <i>null hypothesis</i> is rejected if a suitably-chosen measure lies in a critical region. The critical	D11 46
region is chosen so that if the hypothesis were true, then there would be only a certain chance, often chosen to be 5%, of the measure lying in the critical region. The value at which a critical region starts is known as a	
critical value.	
critical value See critical region.	D11 46
cross multiplication A method of <i>clearing the fractions</i> in an <i>equation</i> that consists of a fraction on each side. The <i>numerator</i> on each side is multiplied by the <i>denominator</i> on the other side and the results are equated.	D14 193 (42)
cube A prism with square cross-section and square sides.	C8 59
cube (of a number) The cube of a number is the number raised to the power 3.	A3 131 (32)
cube root A cube root of a number is a number whose <i>cube</i> is the original number.	A3 151 (32)
cuboid A <i>prism</i> with rectangular cross-section and rectangular sides.	C8 59 (40)
cycle (of a sinusoidal curve) Another name for oscillation.	D14 222
cylinder A three-dimensional shape formed by filling in the space between two parallel congruent <i>circles</i> . The straight line joining the centres of the circles is perpendicular to the circles.	C8 59 (40)
data Data are facts or statistics.	A2 77
data point A point plotted on a <i>scatterplot</i> . Also, another name for a data value.	B6 68
dataset A dataset is a collection of data, usually in tabular form.	A4 183
decagon A polygon with ten sides.	C8 23
decimal places (d.p.) The positions of digits to the right of the decimal point. Also used to indicate the precision of an answer, for example 'to three decimal places (to 3 d.p.)' means 'to three-digit precision after the decimal point'.	A1 19 (28)
degree A degree (indicated by °) is 1/360 th of a full turn. It also means an increment on the Celsius or Fahrenheit temperature scales.	A2 98 and C8 9
denominator The bottom number or expression in a numerical or algebraic fraction.	A1 33 and B5 30 (29, 32)
dependent variable In a practical formula, the subject is often referred to as the dependent variable and the other variable as the independent variable.	B6 67 (35)
depreciation The decline in the value of an item.	D13 135
diameter A <i>chord</i> that passes through the centre of a <i>circle</i> , or the length of such a chord.	C8 54
difference A difference between two numbers is the result of subtracting one from the other, usually the smaller from the larger.	A1 14 (28, 31)

A1 18 (28) **digit** A (decimal) digit is one of the symbols $0, 1, \ldots, 9$. dimensionless quantity A quantity that has no units associated C8 54 with it – that is, a pure number. direct proportion Two quantities x and y are (directly) proportional B6 87, 88 to each other if they are related by an equation of the form y = kx, where k is a non-zero *constant* known as the constant of proportionality. discrete data Data that can take one of a particular set of separated A4 185 values (such as a set of *integers* or the set of shoe sizes). discrete exponential change See exponential change. D13 125, 134 (47) **discriminant** The value $b^2 - 4ac$ is called the discriminant of the C10 157 (44) quadratic expression $ax^2 + bx + c$. divisible Capable of being divided without a remainder. **A3 122** (31) divisor (of an integer) See factor (of an integer). **A3 122** (31) **dotplot** A pictorial representation of a *dataset* using columns of dots D11 9 above a horizontal axis. Each data value is represented by the position of one of the dots along the axis. double inequality A combination of two *inequalities* in the same **A2 106** (30) variable, such as $-2 \le a < 5$ which means $-2 \le a$ and a < 5. doubling time The time it takes for a quantity that grows **D13 166** (48) exponentially to double in size. dropping a perpendicular The process of drawing a line through a D12 76 point in a direction at right angles to a given line is known as dropping a perpendicular from the point to the given line. eliminating (an unknown) The process of combining two or more B7 146 equations to obtain a new equation with fewer unknowns. Two expressions with an equals sign between them. B5 12 **B5 37** (35) equation in one unknown An equation in which a single unknown appears one or more times. equilateral triangle A triangle that has all its sides the same length. C8 19 (38) equivalent (equations) Two equations are said to be equivalent, or B5 42 different forms of the same equation, if one can be rearranged to give the other. equivalent (expressions) Two expressions are said to be equivalent, B5 12 (34) or different forms of the same expression, if they yield a common value for each substitution of the letters. equivalent (fractions) When you multiply or divide the numerator **A1 33** (29) and denominator of a fraction by the same non-zero whole number (or non-zero expression, in the case of an algebraic fraction) you obtain an equivalent fraction. equivalent (ratios) When you multiply or divide each number in a **A3 160** (32) ratio by the same non-zero number you obtain an equivalent ratio. Euler's number $(e \approx 2.718)$ The value b for which the graph of D13 146 $y = b^x$ has a gradient of 1 at (0,1). evaluating (an expression) The process of substituting numbers for B5 11 the letters in an expression and calculating its value. even (integer) An even integer is one that is divisible by 2. A1 44

expanding (an algebraic fraction) The process of dividing each <i>term</i> in the <i>numerator</i> of the <i>algebraic fraction</i> by the <i>denominator</i> .	B5 31 (34)	
expanding the brackets See multiplying out the brackets.	B5 25 (34)	
exponent See index form.	A3 131 (32)	
exponential change A variable y is said to change exponentially with respect to a variable x if the relationship between x and y is given by an equation of the form $y = ab^x$, where a and b are positive constants, with b not equal to 1. If $b > 1$, then y is said to grow exponentially. If $0 < b < 1$	D13 119, 120,	125 (47)
then y is said to decay exponentially. If the change happens in steps (x takes values from a range of equally-spaced numbers, such as the non-negative $integers$), then it is discrete exponential change (also called geometric change). If the change happens continuously (x takes values from an $interval$ of real numbers, such as the non-negative $real\ numbers$), then it is continuous exponential change.		
exponential (growth/decay) curve The graph of an equation of the form $y = ab^x$, where a and b are positive constants, with b not equal to 1. The curve is known as an exponential growth curve if $b > 1$, and as an exponential decay curve if $0 < b < 1$.	D13 126 (48)	
exponential decay See exponential change.	D13 120, 125	
exponential equation An equation in which the unknown is in an exponent, such as $2^{x+1} = 5$.	D13 148 (48)	
exponential function A function whose rule is of the form $y = b^x$ for some positive constant b that is not equal to 1. See also the exponential function.	D13 138 (48)	
exponential growth See exponential change.	D13 119, 125	
exponential model A model based on a formula of the form $y = ab^x$, where a and b are positive constants with b not equal to 1.	D13 126	
exponential regression The process of fitting an <i>exponential curve</i> as closely as possible to a given set of paired data.	D13 129	
expression An algebraic expression, or just expression, is a collection	B5 11 (34, 36)	
of letters, numbers and/or mathematical symbols, arranged in such a way that if numbers are substituted for the letters, then you can work out the		
value of the expression. An expression does not have to contain letters, but the term 'algebraic expression' is usually used only if it does.		88 8
exterior angle (of a polygon) An angle outside the polygon formed by a side and an extended adjacent side.	C8 23	
extrapolation The process of using a set of paired data to estimate a new data point that lies to the left of all the data points given by the paired data, or to the right of all of them.	B6 106	
F angles An informal name for corresponding angles (on parallel lines).	C8 14 (38)	
factor (of an integer) An <i>integer</i> that divides a second integer exactly is called a factor, or divisor, of that second integer. (Sometimes, such as when the natural numbers are being considered, the words 'factor'	A3 122 (31)	
and 'divisor' are used to refer to positive factors only.)		
factor (of a term) If a term can be written in the form something × something (by reordering its letters, factorising its coefficients, and so on), then each 'something' is a factor of the term.	B7 134 (36)	

A3 122 (31) factor pair A pair of integers whose product is equal to a given integer is called a factor pair of that integer. (Sometimes, such as when the natural numbers are being considered, the term 'factor pair' is used to refer to pairs of positive factors only.) factor tree A tree-like diagram illustrating how a factorisation has A3 126 been carried out. factorisation (of an expression) The reverse of multiplying out the **B7 136** (36) brackets. factorisation (of an integer) The process of writing an integer as a A3 127 (31) product of factors that are integers not equal to 1 or -1. (Sometimes, such as when the natural numbers are being considered, the word 'factorisation' is used to refer to products of positive factors only.) finite sequence A sequence with a finite number of terms. C9 75 An equation in which one variable, called the subject of the A2 89 formula, appears by itself on the left-hand side of the equation and not at all on the right-hand side, e.g. y = 3x + 2. The word 'formula' is sometimes used more loosely, to mean the expression on the right-hand side of such an equation, e.g. 3x + 2 is a formula for y', or any equation relating two or more variables, e.g. 'x and y are related by the formula y - 3x = 2'. **fractal** A shape that is irregular at all scales, no matter how closely it is A1 50 viewed. Many fractals can be split into parts, each of which is (at least approximately) a reduced-size copy of the whole. A shape that has this property is said to be self-similar. fraction (algebraic) See algebraic fraction. B5 30 fraction (numerical) A number that describes the relationship **A1 33** (29) between part of something and the whole. A fraction consists of two integers: one, the denominator, indicates how many parts of something make up a whole; the second, the *numerator*, indicates how many of these parts the fraction specifies. free-fall equation An equation relating the distance that an object C10 129 has fallen to the time that it has taken to fall. frequency diagram A diagram that shows the frequencies of D11 21 particular items, values or groups of values. function A rule that takes input values and produces output values. B6 86 general cosine function A function whose rule has the form **D14 222** (50) $y = a\cos(b(x-c)) + d$, for some constants a, b, c, d, with a and b non-zero. general sine function A function whose rule has the form **D14 222** (50) $y = a\sin(b(x-c)) + d$, for some constants a, b, c, d, with a and b non-zero. **geometric change** See exponential change. D13 125 **gradient** The gradient (or slope) of the line through the points (x_1, y_1) B6 69, 70, 78 (35) and (x_2, y_2) is $\frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$. It is a measure of how steep the line is. **graph** A diagram showing the relationship between two variables. **A2 84** and **B6 64–65** (30, 35) Typically the relationship is illustrated by associating each variable with an axis, one horizontal and one vertical. Points are plotted whose horizontal

and vertical *coordinates* correspond to related values of the variables. A smooth curve (or straight line if appropriate) is then often drawn through

the points to indicate the relationship's more general behaviour.

greater than (>) A number is greater than another number if it lies to the right of that number on the <i>number line</i> . For example, $-1 > -3$.	A2 104 (30)	
greatest common divisor (GCD) Another name for highest common factor.	A3 124 (31)	
half-life Another term for halving time, often used in the context of radioactive decay.	D13 167	
halving time The time it takes for a quantity that decays exponentially to halve in size.	D13 167 (48)	
hemisphere Either of the <i>solids</i> obtained by cutting a <i>sphere</i> with a <i>plane</i> through its centre.	C8 62	
heptagon A polygon with seven sides.	C8 23	
hexagon A polygon with six sides.	C8 23	
highest common factor (HCF) (of integers) The highest common factor of two or more <i>integers</i> is the largest integer that is a factor of them all.	A3 124 (31)	
highest common factor (of terms) One <i>common factor</i> of two or more <i>terms</i> is said to be higher than a second common factor if the second is a <i>factor</i> of the first and the first is not a factor of the second. (For example, ab is a higher common factor of the terms a^2b and $2abc$ than the common factor a .) A highest common factor of two or more terms is a common factor of the terms such that no other common factor is higher.	B7 135 (36)	
histogram A diagram that represents a <i>dataset</i> by grouping it into contiguous <i>intervals</i> along a <i>horizontal axis</i> . Each interval forms the <i>base</i> of a <i>rectangle</i> whose area (or height if the intervals are of equal width) is <i>proportional</i> to the frequency (number of occurrences) of data values in the interval.	D11 21	
horizontal axis A horizontal line with a scale that is used to specify the horizontal position of a point.	A2 84 and B6 64 (35)	
horizontal coordinate See coordinates.	A2 85 and B6 64	
horizontal displacement (of a sinusoidal curve) The amount by which the point at $(0,0)$ on the <i>sine curve</i> is displaced to the right when the curve is shifted, stretched and/or compressed to obtain a	D14 226 (50)	
sinusoidal curve.		
horizontal intercept See intercept.	B6 83 (35, 43)	
hypotenuse In a right-angled triangle, the longest side, opposite the <i>right angle</i> , is called the hypotenuse.	C8 43 (39)	
identity An <i>equation</i> that is true for all (appropriate) values of its <i>variables</i> .	B5 12	
improper fraction Another name for top-heavy fraction.	A1 35	
included angle An <i>angle</i> between two <i>adjacent</i> sides of a shape is called the included angle (of the two sides).	C8 32	
included side A side of a shape between two <i>angles</i> is called the included side (of the two angles).	C8 32	
independent events Two events are independent if the occurrence (or not) of one event is not influenced by whether the other occurs.	D11 31	
independent variable See dependent variable.	B6 67	

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A3 131 (32)

A3 131 (32)	index form The result of 'raising a number a to the <i>power</i> n ' is usually written a^n . This is known as index form or index notation. The number a is called the base number or base and n is called the power, index or exponent.
A3 135 (32)	index laws Rules that may be used when working with numbers in index form.
A3 131 (32)	index notation See index form.
A2 104 (30)	inequality Any statement involving one or more of the inequality signs.
A2 104 (30)	inequality sign Any of the four symbols $<$, \le , $>$ and \ge .
A1 50	infinite Endless and without limit.
C9 75	infinite sequence A sequence with an infinite number of terms.
C8 56	inscribe To construct (a shape) within another shape so that it touches but does not cross that other shape.
A1 28	integer Any of the numbers $\dots, -3, -2, -1, 0, 1, 2, 3, \dots$; that is, the negative whole numbers, zero, and the positive whole numbers.
B6 82 (35, 43, 48)	intercept A value on a <i>graph</i> axis scale where the curve (or straight line) meets the axis. An intercept on the <i>horizontal axis</i> is known as a horizontal intercept or <i>x</i> -intercept, and an intercept on the <i>vertical axis</i> is known as a vertical intercept or <i>y</i> -intercept.
C8 17	interior angle (of a polygon) An angle inside the polygon formed by two adjacent sides.
B6 106	interpolation The process of using a set of paired data to estimate a new data point whose horizontal coordinate lies between the horizontal coordinates of two of the data points given by the paired data.
A4 202 (33)	interquartile range (IQR) The difference Q3 – Q1 between the upper quartile (Q3) and the lower quartile (Q1) of a dataset.
A2 105 (30)	interval A section of the number line without any gaps.
D12 69	inverse cosine The inverse cosine of a number x , denoted by $\cos^{-1}(x)$ or $\arccos(x)$, is the angle between 0° and 180° (inclusive) whose <i>cosine</i> is x .
D13 169 (49)	inverse functions Two functions whose rules undo each other's effects.
D13 153	inverse operations Two operations that undo each other's effects.
D12 69	inverse sine The inverse sine of a number x , denoted by $\sin^{-1}(x)$ or $\arcsin(x)$, is the angle between -90° and 90° (inclusive) whose <i>sine</i> is x .
D12 69	inverse tangent The inverse tangent of a number x , denoted by $\tan^{-1}(x)$ or $\arctan(x)$, is the angle between -90° and 90° (exclusive) whose tangent is x .
A3 150	irrational number A real number that is not a rational number, for example $\sqrt{2}$.
C8 19 (38)	isosceles triangle A triangle with just two equal sides.
C8 24 (38)	kite A quadrilateral with two pairs of adjacent equal sides.
A3 121 (31)	least common multiple Another name for lowest common multiple.
B6 105	least squares fit line See regression line.

index See index form.

left-skewed (boxplot) See skewed (boxplot).	D11 15	
less than (<) A number is less than another number if it lies to the left of that number on the <i>number line</i> . For example, $-5 < -2$.	A2 103 (30)	
like terms $Terms$ that are the same except possibly for the <i>coefficients</i> (e.g. $1.4pqr$ and $0.7pqr$ are like terms).	B5 16	
line A straight line that extends infinitely far in both directions. Sometimes used as shorthand for <i>line segment</i> when no confusion can arise.	C8 9	
line of symmetry If a shape looks the same when it is reflected in a (mirror placed on a) <i>line</i> through the shape, then the line is called a line of symmetry, reflection line, mirror line or axis of symmetry.	C8 28	
line segment A finite (unbroken) section of a line.	C8 9	
linear equation in one unknown An equation, such as $3x - 2 = 4(2 + x)$, in which after expanding any brackets or fractions, each term is either a constant or a number times the unknown (in particular, there are no x^2 or x^3 terms).	B5 37 (35)	
linear expression An expression of the form $mx + c$, where m and c are constants with $m \neq 0$ and x is a variable or unknown.	B6 104	
linear function A function with a rule of the form $y = mx + c$, where m and c are constants with $m \neq 0$.	B6 104	
linear regression The process of fitting a straight line as closely as possible to a given set of paired data.	D13 129	
linear relationship Two related quantities are said to have a linear relationship if the graph of one against the other is a straight line.	B6 67	
limit In the context of <i>inequalities</i> , a number that provides a restriction, or limitation, on the value of a <i>variable</i> .	A2 103 (30)	
location A single number that represents an 'average', 'typical' or 'central' value of a <i>dataset</i> .	A4 193 (33)	
logarithm The logarithm to base b of a number x , denoted by $\log_b x$, is the power to which b has to be raised to obtain x . For example, $2^4 = 16$ so $\log_2 16 = 4$.	D13 149 and D13	3 154 (48)
logarithmic function A function whose rule is of the form $y = \log_b x$.	D13 169 (48)	
logarithmic scale (to base 10) A scale (such as the Richter scale) in which each increase by 1 on the scale corresponds to a ten-fold increase	D13 153	
in the quantity. The values on the scale are proportional to the logarithms to base 10 of the quantities that they represent.		
lower quartile (Q1) See quartiles.	A4 202 (33)	
lowest common multiple (LCM) (of integers) The lowest common multiple of two or more <i>integers</i> is the smallest positive integer that is a <i>multiple</i> of them all.	A3 121 (31)	
lowest terms A numerical <i>fraction</i> is in its lowest terms or simplest form when it has been <i>cancelled</i> to give an integer <i>numerator</i> and integer <i>denominator</i> of smallest possible magnitude.	A1 34 (29)	
magnitude (of a number) The value of the number without its sign, if it has one. For example, the magnitude of 3 is 3, and the magnitude of -3 is also 3. The magnitude of a number is often referred to as its size, modulus or absolute value.	B6 74	
manipulating See rearranging (an equation) and rearranging (an expression).	B5 12, 42 (34, 37)

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A2 69 (29)	map scale The relationship between a distance on the map and the corresponding distance on the ground. It is often indicated by a graduated line, or as a ratio such as 1:500 000. See also scale factor (of a map).
A2 66 (30)	mathematical model A collection of assumptions and mathematical statements that attempts to describe how some aspect of the real world behaves, and to make some predictions about its behaviour.
C10 172 (44)	maximisation problem The problem of finding the maximum value of a quantity and the circumstances under which it is obtained.
A4 196 (33)	mean See arithmetic mean.
A4 196 (33)	median When the values in a <i>dataset</i> are arranged in increasing (or decreasing) order, the median is the middle value if the number of values is odd, and the mean of the middle two values if the number of values is even.
C10 179 (44)	minimisation problem The problem of finding the minimum value of a quantity and the circumstances under which it is obtained.
A1 35	mixed number A number that consists of a whole number plus a proper fraction, such as $1\frac{2}{3}$.
A2 79 (30)	modelling cycle The process of designing a <i>mathematical model</i> by clarifying a problem, making assumptions to simplify it, describing it mathematically in order to obtain results, and using the results to refine the assumptions.
B6 74	modulus (of a number) See magnitude (of a number).
C9 106	multiple (of an expression) An expression that has the original expression as a <i>factor</i> .
A3 121	multiple (of a number) A multiple of a number is the result of multiplying it by an <i>integer</i> . For example, $\dots -12, -6, 0, 6, 12, 18, \dots$ are multiples of 6. (Sometimes, such as when the natural numbers are being considered, the word 'multiple' is used to refer to positive multiples only.)
D13 118	multiplication factor Another name for scale factor (of exponential change).
B5 25 (34)	multiplier An expression by which a bracketed expression is multiplied. For example, in $3xy(2+x^2)$ the multiplier is $3xy$.
B5 25 and C9 82 (34, 40)	multiplying out the brackets The process of multiplying terms in brackets by a multiplier or by terms in other brackets to obtain an equivalent expression in which the brackets are not present.
C10 141	n-shaped (parabola) A parabola that is the opposite way up from the graph of $y = x^2$, i.e. its vertex is its highest point.
D13 155	natural logarithm The natural logarithm of a number x is the power to which e has to be raised to obtain x (that is, it is the logarithm of x to base e). The notation 'ln' is usually used in place of 'log $_e$ '.
A1 44	natural number Any one of the counting numbers $1, 2, 3, 4, \ldots$
B6 108	negative correlation The quantities in a set of <i>paired data</i> are said to have a negative correlation if one of the quantities tends to decrease as the other increases. In such cases the <i>correlation coefficient</i> is negative and the <i>regression line</i> has a negative <i>gradient</i> .
C8 60	net (of a solid) A two-dimensional shape that can be folded to obtain the surface of the solid.
C8 23	nonagon A polygon with nine sides.

null hypothesis The assumption that a phenomenon under investigation does not exist.	D11 36
number line A representation of the <i>real numbers</i> on a line in which the numbers become larger from left to right. In particular, all positive numbers lie to the right of zero and all negative numbers lie to the left of zero.	A1 28
numerator The top number or expression in a numerical or algebraic fraction.	A1 33 and B5 30 (29, 32)
oblong number A number given by the expression $n(n+1)$ for some natural number n . Each such number is double a triangular number.	C9 78
obtuse angle An angle greater than 90° and less than 180°.	C8 10
octagon A polygon with eight sides.	C8 23
odd (integer) An odd integer is one that is not divisible by 2.	A1 45
opposite (angle and side) In a triangle, a side and angle are opposite if the side is not one of the two line segments that form the angle.	D12 60
opposite angles (between two lines) Angles between two intersecting lines, that are opposite each other. These angles are equal.	C8 13 (38)
opposite angles (in a quadrilateral) Two angles in a quadrilateral that do not have a side in common.	C8 23 (38)
order (of a rotational symmetry) See rotational symmetry.	C8 27
origin The point with <i>coordinates</i> $(0,0)$.	A2 85
oscillation (of a sinusoidal curve) Any section of the graph of a sinusoidal function whose width on the x-axis is the period of the function.	D14 222
outliers One or more data values in a <i>dataset</i> that are considerably smaller or larger than the others.	A4 189 (33)
paired data Lists of data values for two different variables that occur in pairs, such that for each item in one list there is a corresponding item in the other list. For example, the heights and weights of a group of people can form a set of paired data: each person's height would be paired with their weight.	A4 191
parabola The shape of the <i>graph</i> of any <i>equation</i> of the form $y = ax^2 + bx + c$, where a, b and c are constants with $a \neq 0$.	C10 129 (43)
parabolic (curve) A curve whose shape is all or part of a <i>parabola</i> is said to be parabolic.	C10 129
parallel Two lines in a <i>plane</i> are parallel if they do not cross. This happens if either both the lines are vertical or they both have the same <i>gradient</i> . Similarly, a line and a plane, or two planes, are parallel if they do not cross.	B6 78
parallelogram A quadrilateral with opposite sides equal and parallel.	C8 24 (38, 39)
pentagon A polygon with five sides.	C8 23
percent Means 'per 100' and is denoted by %. For example, $7\% = \frac{7}{100}$.	A1 37 (29)
perfect correlation The quantities in a set of paired data are said to have a perfect correlation if all the data points lie on the regression line. In such cases the correlation coefficient is either 1 or -1 .	B6 108
perfect square A quadratic expression that is equivalent to one of the form $(ax + b)^2$, where a and b are constants. Also, another name for a	C9 95 and A3 153 (40)

 $square\ number.$

C8 48 **perimeter** The boundary of a shape, or the length of the boundary. **period** A function and its graph are periodic, with period p units, if p is D12 95 and D14 222 the smallest positive number for which the shape of the graph repeats itself every p units to the right or left. For example, the sine function has period 360°. C8 47 **perpendicular** A line at right angles to a given line. perpendicular height C8 49 (39) The height of a shape when measured along a line at right angles to a side (or face) of the shape chosen to be the base. A representation of data by means of pictures. For D14 186 example, copies of a picture suggestive of a topic might be stacked one above the other to form the bars of a chart similar to a bar chart, or they may each be labelled by a data item and have a size that represents a value associated with the item, such as its frequency. plane A flat surface that extends infinitely far in all directions. C8 9 plane shape A shape that can be drawn in a plane. C8 48 **point** A point has position but no size. C8 9 **polygon** A plane shape with straight sides. C8 23 polyhedron A solid with flat faces. D14 215 B6 108 positive correlation The quantities in a set of paired data are said to have a positive correlation if one of the quantities tends to increase as the other increases. In such cases the correlation coefficient is positive and the regression line has a positive gradient. To raise a number to a power that is a positive *integer*, multiply **A3 131** (32) it by itself the number of times specified by the power. For example, $2^3 = 2 \times 2 \times 2$. A number can also be raised to a power that is not a positive integer. The meaning of this operation, for powers that are rational numbers, is defined by the index laws $a^0 = 1$, $a^{-n} = 1/a^n$ and $a^{m/n} = (\sqrt[n]{a})^m$. See also index form. A1 23 precision (of an answer) How many significant figures the answer is stated to. precision (of a set of measurements) How close the A4 213 measurements in a set of repeated measurements are to each other. A4 183 **primary data** Data that you collect yourself. A3 124 **prime** (number) A natural number that has exactly two factors (itself and 1). **prime factorisation** The prime factorisation of a *natural number* is A3 128 (31) the *product* of prime *factors* that is equal to it. C8 58 (40) A three-dimensional shape formed by filling in the space between two parallel *congruent* polygons. These polygons are faces of the prism and they are congruent to all parallel cross-sections through the prism. The edges joining the vertices of one of the polygons to the matching vertices of the other polygon are perpendicular to the polygons. **probability** A measure of how likely something is to occur. A D14 181 (49) probability can be expressed as a fraction, as a decimal, as a percentage or in the form of an x in y chance. For example, $\frac{1}{200}$, 0.005 and 0.5% are all ways to describe a 1 in 200 chance. **product** The product of two or more numbers is the result of **A1 14** (28, 31) multiplying them.

projectile An object that is propelled through space by a force that ceases after launch.	C10 132
proof A demonstration that a piece of mathematics always works.	B5 7
proper fraction A numerical fraction in which the magnitude of the numerator is smaller than that of the denominator, such as $\frac{2}{3}$.	A1 35
proportional See direct proportion.	B6 87, 88
pseudo-random (numbers) Random numbers generated by a computer algorithm.	D11 29
Pythagorean triple Three whole numbers such that the square of one of them is equal to the <i>sum</i> of the squares of the other two. For example, the numbers 3, 4, and 5 form a Pythagorean triple, since $5^2 = 3^2 + 4^2$.	C8 46 (39)
quadrant The x- and y-axes divide the plane into four regions known as quadrants. The quadrant between the positive x- and y-axes is called the first quadrant, followed (anticlockwise) by the second, third and fourth quadrants.	D12 88
quadratic (expression) An expression of the form $ax^2 + bx + c$, where a , b and c are constants with $a \neq 0$, is called a quadratic expression in x , or a quadratic in x , or just a quadratic.	C9 87 (41)
quadratic equation Any equation that can be expressed in the form $ax^2 + bx + c = 0$ (by rearranging if necessary) is called a quadratic equation in x . In this equation, x is an unknown, and a , b and c are constants with $a \neq 0$.	C9 88 (42, 43, 44)
quadratic formula A formula that gives the solutions of a quadratic equation.	C10 152 (43)
quadratic function A function whose rule is of the form $y = ax^2 + bx + c$, where a, b and c are constants with $a \neq 0$.	C10 138 (43)
quadratic model A mathematical model based on a formula of the form $y = ax^2 + bx + c$, where a, b and c are constants with $a \neq 0$.	C10 130
quadrilateral A polygon with four sides.	C8 23
quartiles When the values in a <i>dataset</i> are arranged in ascending order, the lower quartile is the <i>median</i> of the values in the lower half of the dataset, and the upper quartile is the median of the values in the upper half of the dataset (with, in each case, the middle value thrown out if the number of values is odd).	A4 202 (33)
quotient A quotient of two numbers is the result of dividing one by the other.	A1 14 (28, 31)
radian A unit used to measure angle. One radian is the angle subtended at the centre of a circle by an arc that is the same length as the radius. A full turn is 2π radians.	D12 102 (46)
radius A line segment from the centre to the circumference of a circle, or the length of such a line segment.	C8 54 (39)
radius (of a circular arc) The radius of the circle on whose circumference the arc lies.	D12 104
random numbers Short for <i>uniform random numbers</i> (unless the context indicates that the numbers are not equally likely to occur).	D11 28 (45)
range The range of a <i>dataset</i> is the <i>difference</i> between its largest and smallest values.	A4 200

B6 79 rate of change The gradient of a straight-line graph tells you the amount by which the variable on the vertical axis increases when the variable on the horizontal axis increases by one unit. This increase in the first variable (which is actually a decrease if negative) is known as the rate of change of the first variable with respect to the second. A ratio of two or more quantities specifies how many parts of each **A3 159** (32) quantity there are. For example, the ratio 1:2:4 (read 1 to 2 to 4) means that there is 1 part of the first quantity for every 2 parts of the second quantity and every 4 parts of the third quantity. A ratio of two quantities is sometimes written as a fraction or decimal. For example, 3:2 can be written as $\frac{3}{2}$ or as 1.5. rational number A number that can be written as an integer divided A3 136 by an integer. A3 150 real line Another name for the *number line*, since each point on the number line represents a real number. A3 150 real numbers The set of all the rational numbers together with all the irrational numbers. Each real number is represented by a point on the number line. rearranging (an equation) Rearranging (or manipulating) an **B5 42** (35) equation is the process of: doing the same thing to both sides of the equation; rearranging the expressions in the equation; or swapping the sides of the equation. rearranging (an expression) The process of writing an expression **B5 12** (34) in a different way to obtain an equivalent expression is known as rearranging, manipulating or rewriting the expression. **reciprocal** The reciprocal of a number is 1 divided by the number. A3 142 rectangle A quadrilateral with four right angles. C8 24 (39) recurring decimal A decimal number with a block of one or more A3 137 digits after the decimal point that repeats indefinitely. C8 28 reflection line See line of symmetry. C8 10 reflex angle An angle greater than 180° and less than 360°. B6 105 regression line The straight line on a scatterplot of paired data that 'best' fits the data. Other names for this line include least squares fit line, best fit line and trend line. regular (polygon) A polygon with equal sides and equal interior C8 25 angles. A1 39 relative comparison A relative comparison is one in which proportions are used, whereas an absolute comparison is one in which differences are used. For example, if the values of A and B are 5 and 10, respectively, then the statement 'B is twice as big as A' is a relative comparison, whereas the statement 'B is 5 units larger than A' is an absolute comparison. **B5 25** (34) removing the brackets See multiplying out the brackets. C9 96 (44) repeated solution If the two solutions of a quadratic equation are the same, then the equation is said to have a repeated solution. A1 48 See theorem. rewriting an expression See rearranging an expression. B5 12 (34)

rhombus A parallelogram with four equal sides.	C8 24	
right angle An angle of 90°.	C8 10	
right-angled triangle A triangle in which one angle is equal to 90°.	C8 19	
right-skewed (boxplot) See skewed (boxplot).	D11 15	
rise Given two points (x_1, y_1) and (x_2, y_2) , with $x_1 < x_2$, the rise is $y_2 - y_1$.	B6 70 (35)	
risk The probability of an undesirable event occurring.	D14 181	
root mean squared deviation (RMS) Another name for standard deviation.	A4 205 (34)	
rotational symmetry A shape has rotational symmetry if it can be rotated through a fixed <i>angle</i> (less than a full turn) about a fixed <i>point</i> to produce a rotated shape that looks the same as the original shape. If there are, say, three positions in which the shape looks the same, then the shape is said to have rotational symmetry of order 3, or three-fold rotational	C8 27	
symmetry.	A 1 04	
rounding error An error in an answer resulting from rounding performed at an earlier step of the calculation.	A1 24	
run Given two points (x_1, y_1) and (x_2, y_2) , with $x_1 < x_2$, the run is $x_2 - x_1$.	B6 70 (35)	
satisfying (an equation) See solution (of an equation in one unknown).	B5 36 (35)	
satisfying (an inequality) A value of a <i>variable</i> for which an <i>inequality</i> is true is said to satisfy the inequality.	A2 105 (30)	
satisfying (simultaneous equations) See solution (of simultaneous equations).	B7 146 (37)	
scale factor (of exponential change) If a quantity is subject to discrete exponential change, then from an initial starting number each subsequent value is obtained by multiplying its predecessor by a constant. This constant is known as the scale factor. More generally, if a variable y changes exponentially with respect to x (continuously or discretely), then whenever the value of x increases by a fixed number of units, the value of y is multiplied by a constant. This constant is called the scale factor over that number of units. In particular, if the relationship between x and y is given by the equation $y = ab^x$, then b is the scale factor over 1 unit (and a is the starting number).	D13 118, 121,	134, 137 (47)
scale factor (of an image) The number by which distances on an image are multiplied when the image is enlarged or reduced.	A3 165	
scale factor (of a map) The number by which a distance on the map has to be multiplied to obtain the actual distance on the ground.	A2 69 (29)	
scalene triangle A triangle all of whose sides are different lengths.	C8 20	
scatterplot A graph on which paired data are plotted.	B6 68	
scientific notation A notation in which a number is written as a decimal number between 1 and 10 (including 1, but excluding 10), multiplied by a power of 10; for example, 1.92×10^{-2} and 9.994×10^{30} .	A3 145 (32)	
secondary data Existing <i>data</i> that you can use or adapt for your purpose.	A4 183	
sector The shape enclosed by an <i>arc of a circle</i> together with the two <i>radii</i> from the endpoints of the arc.	C8 54	
		22

C8 54

C8 55

D12 86 (46)

C9 75 (40)

D12 87

angles correspond to clockwise rotations. significant figures (s.f.) The first significant figure in a number is **A1 20** (28) the first non-zero digit when reading the number from left to right, the second significant figure is the digit immediately to the right of this digit and so on. similar Geometric figures that have the same shape (flipped if C8 29 (39) necessary), but not necessarily the same size, are said to be similar. simplest form (of a fraction) Another name for *lowest terms*. A1 34 (29) simplest form (of a ratio) A ratio is in its simplest form if the A3 160 (32) numbers in the ratio are whole numbers with no positive common factor simplifying an expression or equation The process of rearranging **B5** 12 (34) an expression or equation to make it simpler. simulation (using random numbers) The process of using D11 31 random numbers to investigate statistical features that may or may not occur by chance. simultaneous equations Two or more equations that apply to the **B7** 143 (37) unknowns simultaneously. The sine of an angle θ , written $\sin \theta$, is the y-coordinate of the D12 61, 90 (45) point obtained by rotating the point (1,0) about the origin through the angle θ . For an acute angle θ in a right-angled triangle, $\sin \theta$ is equal to the length of the side opposite θ divided by the length of the hypotenuse. sine curve The graph of the *sine* function. D12 94 Sine Rule A rule for solving a triangle in which a side and its opposite D12 77 (45) angle, together with at least one other angle or side, are known. sinusoidal curve A curve that can be obtained by shifting, stretching **D14 222** (50) or compressing the graph of the *sine* function horizontally or vertically. sinusoidal function A function whose graph is a sinusoidal curve. All **D14 222** (50) general sine functions and all general cosine functions are sinusoidal functions. Moreover, every sinusoidal function can be expressed as either a general sine function or as a general cosine function. size (of a number) See magnitude (of a number). B6 74 skewed (boxplot) A boxplot is left-skewed if the data values are more D11 15 sparsely spread at the left and more densely concentrated at the right of the boxplot. Likewise, a boxplot is right-skewed if the data values are more sparsely spread at the right and more densely concentrated at the left of the boxplot. The distance from the apex of the cone to a slant height (of a cone) C8 59 (40) point on the circumference of its base.

segment The shape enclosed by an arc of a circle and the chord

an arc from one end of the diameter to the other.

sequence A list, usually of numbers.

semicircle The shape enclosed by a diameter of a circle, together with

semi-perimeter (of a shape) Half of the *perimeter* of the shape.

sign of an angle This indicates the direction of rotation about a

point. Positive angles correspond to anticlockwise rotations, and negative

joining the ends of the arc.

slope See gradient.	B6 69, 70, 78 (35)
solid Another name for a three-dimensional shape.	C8 58 (40)
solution (of an equation in one unknown) Any value of the <i>unknown</i> that makes the two sides of the <i>equation</i> equal is said to satisfy the equation and is called a solution of the equation. The process of finding such a solution is known as solving the equation.	B5 36 (35)
solution (of simultaneous equations) Values of the <i>unknowns</i> that simultaneously satisfy all the equations are together called a solution of the <i>simultaneous equations</i> . Such a solution is said to satisfy the equations. The process of finding a solution is known as solving the equations.	B7 146 (37)
solving (an equation) See solution (of an equation in one unknown).	B5 36 (35)
solving (an inequality) The process of finding all the numbers that satisfy an <i>inequality</i> .	B7 164
solving (simultaneous equations) See solution (of simultaneous equations).	B7 146 (37)
solving a triangle The process of calculating unknown lengths or angles in a triangle.	D12 75, 83 (46)
speed A measure of how far an object travels in a particular period of time. See also <i>average speed</i> .	A2 72 (30)
sphere A three-dimensional shape whose boundary consists of all points that are a fixed distance from a fixed point called the centre of the sphere.	C8 61 (40)
spread (of a dataset) How widely the values in the dataset are distributed.	A4 200 (34)
spurious precision The display of values to a greater-than-warranted number of <i>significant figures</i> .	A4 189 (33)
square A quadrilateral with four equal sides and four right angles.	C8 24
square (of a number) The square of a number is the result of multiplying it by itself.	A3 131
square numbers The numbers $1, 4, 9, 16, \ldots$, obtained by multiplying each <i>natural number</i> by itself.	A1 45
square root A square root of a number is a number that when multiplied by itself gives the original number.	A3 151 (32)
standard deviation (SD) The standard deviation of a set of values is the <i>square root</i> of the <i>mean</i> of the squares of the deviations, where the deviations are the <i>differences</i> of each value from the mean. (Sometimes a slightly different definition is used – see page 34.)	A4 205 (34)
standard form Another name for scientific notation.	A3 145 (32)
starting number See scale factor (of exponential change).	D13 118 (47)
straight angle An angle of 180°.	C8 10 (38)
strict inequality A statement involving one or more of the inequality signs $<$ or $>$, but not \le or \ge .	A2 104
subject (of a formula) See formula.	A2 89
subscript Characters such as the 1 and the 2 in $x_2 - x_1$ are known as subscripts; they are smaller and set slightly lower than normal. They are often used to distinguish distinct but related variables.	B6 76

substituting The process of replacing variables in an expression or A2 89 equation with numerical values. subtended (angle) The angle between the line segments that join a D12 102 point to each end of an arc is said to be the angle subtended by the arc at the point. The result of adding together two or more numbers. sum **A1 14** (28, 31) surd A numerical expression containing one or more *irrational* roots of **A3 153** (32) numbers. surface area (of a solid) The area of the *solid's* surface. C8 60 (40) tangent The tangent of an angle θ , written tan θ , is defined by D12 61, 91, 92 $\tan \theta = \sin \theta / \cos \theta$, provided that $\cos \theta \neq 0$. It is the y-coordinate of the point where the line x = 1 meets the line obtained by rotating the x-axis through the angle θ about the *origin*. For an acute angle θ in a right-angled triangle, $\tan \theta$ is equal to the length of the side opposite θ divided by the length of the side adjacent to θ . term (of an expression) In an expression formed by adding or B5 13 subtracting a list of items, each item is called a term of the expression. A sign (plus or minus) at the start of a term is part of the term. term (of a sequence) An entry (usually a number) in the sequence. C9 75 (40) A decimal number that has only a finite terminating decimal A3 137 number of digits after the decimal point. the exponential function The function whose rule is $y = e^x$. D13 147 theorem A mathematical statement that has been proved is called a A1 48 theorem or result. three-dimensional (shape) A shape that extends in three mutually C8 58 (40) perpendicular directions. In a sketch: width is extent across the page; height is extent up and down the page; and depth is extent into the page (using perspective). top-heavy fraction A numerical fraction in which the magnitude of A1 35 the numerator is larger than that of the denominator, such as $\frac{5}{3}$. The path that a projectile follows. trajectory C10 132 A quadrilateral with one pair of opposite sides parallel. trapezium C8 24 (39) trend line See regression line. B6 105 trial A single experiment or observation for which a number of D11 28 outcomes are possible, but only one can occur at a time. For example, the tossing of a coin is a trial with an outcome of either 'head' or 'tail'. trial and improvement A way of 'homing in' on the solution of an D13 132 equation by repeatedly trying values to see whether, at each stage, a larger

or a smaller value would improve the approximation.

second row, and so on, up to n dots in the final row.

triangular number A number given by the expression $\frac{1}{2}n(n+1)$ for

some $natural\ number\ n$. Each such number is the number of dots that can be arranged in a triangular shape with 1 dot in the first row, 2 dots in the

C9 74

triangular prism A prism with triangular cross-section.	C8 58 (40)
trigonometric function A function whose rule takes an angle θ a input, and outputs one of the trigonometric values associated with that angle, such as $\sin \theta$, $\cos \theta$ or $\tan \theta$.	* * *
trigonometric ratio (of an angle) The ratio of two sides of a right-angled triangle that contains the angle.	D12 61 (45)
trigonometry The branch of mathematics that is concerned with methods of using triangles to find unknown lengths and angles.	D12 58
two-dimensional (shape) A shape that extends in two directions	s. C8 58
two-sample A two-sample dataset is one that consists of two sets of values of the same <i>variable</i> , allowing the two samples to be compared.	A4 191
u-shaped (parabola) A parabola that is the same way up as the graph of $y = x^2$, i.e. its vertex is its lowest point.	C10 141
uniform random numbers A sequence of numbers, each of whice selected uniformly (that is, with equal chance) and <i>independently</i> of its predecessors.	
unit circle The circle with radius 1 centred on the origin.	D12 88
unknown A letter that represents a particular, though possibly unknown, number.	B5 13
upper quartile (Q3) See quartiles.	A4 202 (33)
variable A letter used to represent different numbers.	A2 89
variance The square of the standard deviation.	A4 205 (34)
vertex A point where two line segments meet.	C8 9
vertex (of a parabola) The point at which the parabola intersect axis of symmetry.	s its C10 138 (43)
vertical axis A vertical line with a scale that is used to specify the vertical position of a point.	A2 84 and B6 64 (35)
vertical coordinate See coordinates.	A2 85 and B6 64
vertical displacement (of a sinusoidal curve) The mean of the maximum and minimum values of the curve.	the D14 226 (50)
vertical intercept See intercept.	B6 83 (35, 43, 48)
volume (of a solid) The amount of space that the solid occupies.	C8 60 (40)
whiskers See boxplot.	D11 11 (45)
X angles An informal name for opposite angles between two lines.	C8 13 (38)
x-intercept A value on a graph's x-axis scale where the graph cross or touches the axis, i.e. a value of x for which $y = 0$.	ses B6 82 and C10 143
y-intercept A value on a graph's y-axis scale where the graph cross or touches the axis, i.e. a value of y for which $x = 0$.	ses B6 82 and C10 143
Z angles An informal name for alternate angles.	C8 14 (38)

4 Key skills and results

The following key skills and results have been collected from the units of MU123. They are usually listed in the order in which they occur in the module, each with a reference in the margin to its location.

Unit 1: Starting points

Book A, Unit 1, page 13 Using the BIDMAS rules

Carry out mathematical operations in the following order.

- B brackets
- I indices (powers and roots)
- D divisionsM multiplications} same precedence
- M multiplications
 A additions
 Same precedence
- A additions
 S subtractions same precedence

When operations have the same precedence, work from left to right.

Book A, Unit 1, Example 2, page 16

Converting units

To convert from one unit to another, first find out how many of the smaller units are equivalent to one of the larger units.

- To convert to the smaller unit, multiply by this number.
- To convert to the larger unit, divide by this number.

Rounding a number

To avoid rounding errors, use full calculator precision throughout a calculation and round only the final answer.

- Look at the digit immediately after where you want to round.
- Round up if this digit is 5 or more, and down otherwise.

For example, 2.3971 = 2.40 (to 2 d.p.) and 36.7972 = 36.80 (to 4 s.f.).

nd 50 64 (39)

Book A, Unit 1, page 19

Book A, Unit 1, page 29

Adding and subtracting negative numbers

- Adding a negative number is the same as subtracting the corresponding positive number. For example, 5 + (-2) = 5 2 = 3.
- Subtracting a negative number is the same as adding the corresponding positive number. For example, 5 (-2) = 5 + 2 = 7.

Book A, Unit 1, page 31

The following table might help you to remember these rules:

Multiplying and dividing negative numbers

When two numbers are multiplied or divided:

- If the signs are different, then the answer is negative. For example, $9 \div (-3) = -3$ and $(-3) \times 7 = -21$.
- If the signs are the same, then the answer is positive. For example, $-9 \div (-3) = 3$ and $3 \times 7 = 21$.

Writing a fraction in its simplest form

Calculating a fraction of a quantity

Keep cancelling the fraction until it is no longer possible to exactly divide both the numerator and the denominator by the same whole number (other than 1). The result will be an equivalent fraction in simplest form.

Book A, Unit 1, page 34 $\frac{12}{30}^{4} = \frac{4}{5}$ $\frac{24}{30} = \frac{4}{5}$

Book A, Unit 1, Example 10, pages 35–36

Book A, Unit 1, page 37

Multiply the fraction by the quantity. pages 35-For example, $\frac{5}{8}$ of $20 = \frac{5}{8} \times 20 = 5 \div 8 \times 20 = 12.5$.

Converting a percentage to a fraction or decimal

First write the percentage in the form of a fraction with denominator 100, then simplify to obtain a fraction, or divide out to obtain a decimal.

For example,
$$45\% = \frac{45}{100} = \frac{9}{20}$$
 and $45\% = \frac{45}{100} = 45 \div 100 = 0.45$.

Book A, Unit 1, page 37

Converting a fraction or decimal to a percentage

Multiply the fraction or decimal by 100% (= 1).

For example,
$$\frac{2}{5} = \frac{2}{5} \times 100\% = 40\%$$
 and $0.015 = 0.015 \times 100\% = 1.5\%$.

Book A, Unit 1, page 38

Expressing a number as a percentage of another number

Calculate $\frac{\text{first number}}{\text{second number}} \times 100\%.$

Book A, Unit 1, page 40

Calculating a percentage of a quantity

Change the percentage to a fraction or a decimal, and multiply by the quantity.

For example, 2.5% of $450 = \frac{2.5}{100} \times 450 = 0.025 \times 450 = 11.25$.

Book A, Unit 1, page 40

Calculating a percentage increase or decrease

 $\label{eq:calculate} \mbox{Calculate} \quad \frac{\mbox{actual increase or decrease}}{\mbox{original value}} \times 100\%.$

Calculating the value resulting from a percentage change

Change 100% by the required percentage and multiply the resulting adjusted percentage by the value. For example, if 599 is decreased by 15%, the new value is 85% of $599 = 0.85 \times 599 = 509.15$. If 800 is increased by 5%, the new value is 105% of $800 = 1.05 \times 800 = 840$.

Book A, Unit 1, pages 41–42

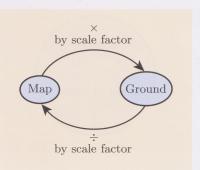
Unit 2: Mathematical models

Using the scale factor of a map to convert distances

If not already known, deduce the scale factor from the map scale. (For instance, the scale $1:500\,000$ means that the scale factor is $500\,000$, as does the scale '2 cm represents $10\,\mathrm{km}$ '.)

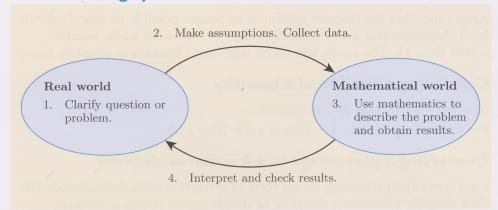
- To calculate a ground distance: multiply the map distance by the scale factor of the map and express the result in the required units.
- To calculate a map distance: divide the ground distance by the scale factor of the map and express the result in the required units.

Book A, Unit 2, Example 2, pages 70–71



Book A, Unit 2, page 79

The modelling cycle



Book A, Unit 2, page 84

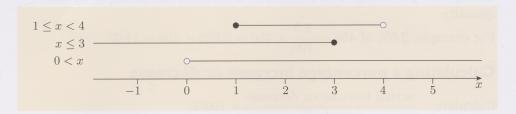
Drawing a graph or chart based on data

- Include a clear title and the source of the data.
- Label the axes with the names of the quantities and (if applicable) the units.
- Mark the scales clearly, choosing scales that are easy to interpret and that make good use of the space available.

Book A, Unit 2, Example 15, page 106

Using inequalities to specify the values taken by a variable

In many cases the values taken by a variable form an interval. The following examples, concerning a variable x, illustrate how inequalities are used to describe such intervals.



- An inequality such as x < 4 constrains x to lie to the left of a limit (here 4).
- An inequality such as 0 < x constrains x to lie to the right of a limit (here 0).
- To include the limit in the constraint, use an inequality that is not strict (\leq or \geq).
- Diagrammatically, inclusion of a point is indicated by a solid circle and exclusion is indicated by an empty circle.

Book A, Unit 2, page 109

distance speed time

Using the speed, distance and time formulas

The three formulas relating the distance, average speed and time for a journey can all be recalled from the diagram in the margin. They are:

$$speed = \frac{distance}{time}, \quad time = \frac{distance}{speed}, \quad distance = speed \times time.$$

Unit 3: Numbers

Finding the factors of a natural number (factorisation)

Book A, Unit 3, page 122

- 1. Try the numbers $1, 2, 3, 4, \ldots$ in turn. Whenever you find a factor, write down the other factor in the factor pair.
- 2. Stop when you get a factor pair that you have already.

Testing for divisibility

Book A, Unit 3, page 123

A natural number is divisible by

- 2 if it ends in 0, 2, 4, 6, or 8
- 3 if its digits add up to a multiple of 3
- 5 if it ends in 0 or 5
- 9 if its digits add up to a multiple of 9.

If a number does not satisfy a test above, then it is not divisible by the specified number.

The fundamental theorem of arithmetic

Every natural number greater than 1 can be written as a product of prime numbers in just one way (except that the order of the primes in the product can be changed).

Book A, Unit 3, page 128

Obtaining a prime factorisation of a composite number

Proceed step by step as follows:

- 1. Divide the number by its smallest prime factor.
- 2. Decide whether the result is a prime; if not, divide it by its smallest prime factor.
- 3. Repeat the previous step until the result produced is a prime.

The example in the margin shows a handy way to record the steps. The required factorisation appears on the final line.

Book A, Unit 3, pages 128-129

$$252 = 2 \times 126$$

$$= 2 \times 2 \times 63$$

$$= 2 \times 2 \times 3 \times 21$$

$$= 2 \times 2 \times 3 \times 3 \times 7$$

$$= 2^{2} \times 3^{2} \times 7.$$

Book A, Unit 3, page 130

Finding the LCM or HCF of two or more natural numbers

- Find the prime factorisations of the numbers.
- To find the LCM, multiply together the highest power of each prime factor occurring in any of the numbers.
- To find the HCF, multiply together the lowest power of each prime factor common to all the numbers.

Adding or subtracting fractions

- 1. Make sure that the denominators are the same. (You may need to use a common denominator to write each fraction as an appropriate equivalent fraction.)
- 2. Add or subtract the numerators.
- 3. Write the answer in its simplest form.

Multiplying fractions

Book A, Unit 3, page 140

Book A, Unit 3, page 139

Multiply the numerators together and multiply the denominators together. Write the answer in its simplest form.

Book A, Unit 3, page 142

Dividing by a fraction

Multiply by its reciprocal (obtained by swapping over its numerator and denominator). Write the answer in its simplest form.

Book A, Unit 3, page 145

Expressing a number in scientific notation

- 1. Place a decimal point between the first and second significant digits to give a number between 1 and 10.
- Count to find the power of 10 by which this number should be multiplied (or divided) to restore it to the original number.

Book A, Unit 3, pages 153–156

Simplifying surds

- Simplify roots of integers with square factors (e.g. $\sqrt{12} = 2\sqrt{3}$).
- Simplify products and quotients of roots (e.g. $\sqrt{15}/\sqrt{3} = \sqrt{5}$).
- Add or subtract (multiples of) roots that are the same (e.g. $\sqrt{12} + 3\sqrt{3} = 5\sqrt{3}$).

Book A, Unit 3, page 159

Here the numbers must be appropriate for the operations. For example, you cannot divide by zero, or take a square root of

a negative number.

Index laws (rules for powers)

$$a^{m} \times a^{n} = a^{m+n}, \qquad \frac{a^{m}}{a^{n}} = a^{m-n}, \qquad (a^{m})^{n} = a^{mn}$$
$$(a \times b)^{n} = a^{n} \times b^{n}, \qquad \left(\frac{a}{b}\right)^{n} = \frac{a^{n}}{b^{n}}$$
$$a^{0} = 1, \qquad a^{-n} = \frac{1}{a^{n}}$$

 $a^{\frac{1}{n}} = \sqrt[n]{a}, \qquad a^{\frac{m}{n}} = (\sqrt[n]{a})^m$

Book A, Unit 3, page 160

Finding a ratio equivalent to a given ratio

Multiply or divide each number in the ratio by the same non-zero number. If possible, a ratio is usually written in its simplest form, where the numbers are integers without a common factor greater than 1.

Book A, Unit 3, Activity 37, page 161

Comparing ratios of two numbers

Write each ratio in the form of a fraction (or in the form 'number: 1') and compare the fractions (or the numbers to the left of the colons).

Book A, Unit 3, Example 16, page 161

Finding an approximate ratio

- 1. Write the ratio in the form 'number: 1'.
- 2. Replace the number by a simple fraction that approximates the number.
- 3. Simplify the resulting ratio.

Book A, Unit 3, Example 17, page 162

Dividing a quantity in a ratio

- 1. Calculate the sum of the numbers in the ratio.
- To find the portion of the quantity corresponding to each ratio number, divide the number by the sum and multiply by the quantity.

For example, since 5+2+3=10, the ratio 5:2:3 divides 1250 into $\frac{5}{10} \times 1250 = 625$, $\frac{2}{10} \times 1250 = 250$ and $\frac{3}{10} \times 1250 = 375$, respectively.

Unit 4: Statistical summaries

The four stages of a statistical investigation (PCAI)

There are four clearly identifiable stages in most statistical investigations, which can be summarised as follows.

Stage 1 Pose a question

Stage 2 Collect relevant data

Stage 3 Analyse the data

Stage 4 Interpret the results

The problem starts in the real world and is resolved by making a journey into the statistical world and back again. Complete resolution of the problem might require several trips around the cycle.

Scanning a dataset

Before performing a detailed analysis of any dataset, it is advisable to examine the values to see if any patterns or anomalies stand out. For example, you might look for

- missing data
- spurious precision
- dubious data, perhaps caused by a misplaced decimal point
- coded values, perhaps to signal 'value missing'
- constraints such as that the data ought to lie between 0 and 100
- the presence of outliers.

Finding the mean of a dataset

To find the mean of a set of numbers, add all the numbers together and divide by however many numbers there are in the set.

Finding the median of a dataset

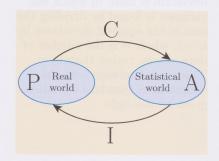
To find the median of a set of numbers:

- Sort the data into increasing (or decreasing) order.
- If there is an odd number of data values, the median is the middle value.
- If there is an even number of data values, the median is the mean of the middle two values.

Finding the quartiles and the interquartile range of a dataset

- 1. Arrange the dataset in increasing order.
- 2. Next:
 - (a) If there is an even number of data values, then the lower quartile (Q1) is the median of the lower half of the dataset, and the upper quartile (Q3) is the median of the upper half of the dataset.
 - (b) If there is an odd number of data values, throw out the middle data value (which of course has the median value of the dataset). Then the lower quartile (Q1) is the median of the lower half of the new dataset, and the upper quartile (Q3) is the median of the upper half of the new dataset.
- 3. The interquartile range (IQR) is Q3 Q1.

Book A, Unit 4, page 180



Book A, Unit 4, page 195

Book A, Unit 4, page 196

Book A, Unit 4, page 197

Book A, Unit 4, page 204

Book A, Unit 4, page 205

In some circumstances a slightly different definition of standard deviation is used, in which the variance is found by dividing the sum of the squared deviations by n-1, where n is the number of data values, rather than by n. This module always uses the definition of standard deviation given on the right.

Book B, Unit 5, page 24

Book B, Unit 5, page 23

Book B, Unit 5, page 22

Remember: a plus times a plus gives a plus; a plus times a minus gives a minus. Also: adding/subtracting the negative of something is the same as subtracting/adding the something.

Book B, Unit 5, page 26

Book B, Unit 5, page 29

Book B, Unit 5, page 31

Finding the standard deviation of a dataset

- 1. Find the mean of the dataset.
- 2. Find the difference of each data value from the mean these are the 'deviations', often labelled the d values.
- 3. Square each deviation this gives the d^2 values.
- 4. Find the mean of these squared deviations this number is the 'mean squared deviation', better known as the variance.
- 5. Find the square root of the variance to get the 'root mean squared deviation' that is, the standard deviation.

Unit 5: Algebra

Simplifying an expression

- 1. Identify the terms.
- 2. Simplify each term, remembering to include the sign (plus or minus) at the start of each term. (If the term includes brackets or algebraic fractions, consider also whether to multiply out the brackets or expand the fractions, but remember to simplify any resulting new terms.)
- 3. Collect any like terms.

Identifying the terms in an expression

Use the fact that each term after the first starts with a plus or minus sign that is not inside brackets. Thus $5ab + 6c^2g\sqrt{2} - (-4d) + (-a)(-3+x)d$ has terms +5ab, $+6c^2g\sqrt{2}$, -(-4d) and +(-a)(-3+x)d.

Simplifying a term in an expression

- 1. Find the overall sign and write it at the front.
- 2. Simplify the rest of the coefficient and write it next.
- 3. Write any remaining parts of the term in some appropriate order; for example, letters are usually ordered alphabetically. Use index notation to avoid writing letters (or other parts) more than once.

For example, $-(-2pq) \times (-3qp^2) = -6p^3q^2$.

Multiplying out brackets

Multiply each term inside the brackets by the multiplier. Simplify each product as you multiply out. For example, $2a(3a + 2b) = 6a^2 + 4ab$.

See the Unit 9 entries for how to multiply out expressions with more than one pair of brackets, such as (a + b)(c + d).

Removing brackets with a plus or minus sign in front

- If the sign is plus, keep the sign of each term inside the brackets the same. For example, $+(a^2+3ab-d)=+a^2+3ab-d$.
- If the sign is minus, change the sign of each term inside the brackets. For example, $-(a^2 + 3ab d) = -a^2 3ab + d$.

Expanding an algebraic fraction

Divide each term of the numerator by the denominator. Simplify each quotient as you divide through. For example, $\frac{10x + x^2 - 8}{x} = 10 + x - \frac{8}{x}$.

Collecting like terms

Replace each group of like terms by a single term whose coefficient is the sum of the coefficients of the terms in the group. For example, a + 5xy + a - 2yx - 2a = 3xy.

Book B, Unit 5, page 17

Solving a linear equation in one unknown

Carry out a sequence of steps. In each step, do one of the following:

- do the same thing to both sides
- simplify one side or both sides
- swap the sides.

Aim to do the following, in order.

- 1. Clear any fractions and multiply out any brackets. To clear fractions, multiply both sides by a suitable number.
- 2. Add or subtract terms on both sides to obtain an equation of the form

$$(a \text{ number}) \times (b \text{ the unknown}) = (a \text{ number})$$

3. Divide both sides by the coefficient of the unknown.

Book B, Unit 5, page 45

Remember that you have to do the same thing to the *whole* of each side, namely one of:

- o add something
- o subtract something
- o multiply by something
- divide by something non-zero.

Unit 6: Graphs

Plotting the graph of a formula

- Construct a table with some values of the independent variable in the first row and corresponding values of the dependent variable in the second row.
- Put the independent variable on the horizontal axis and the dependent variable on the vertical axis, and select a scale that covers all the values in the table.
- Plot the points whose coordinates form the columns of the table and, if appropriate, join them with a straight line or a smooth curve.

Book B, Unit 6, Example 1, page 65; also page 67

Finding the gradient (slope) of a straight line

Choose two points (x_1, y_1) and (x_2, y_2) on the line. Then use

gradient =
$$\frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$
.

The gradient of a vertical line is undefined.

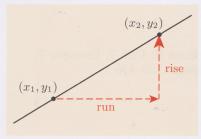
A line that slopes up from left to right has a positive gradient.

A line that slopes down from left to right has a negative gradient.

A horizontal line has gradient 0.

If the axes have units that are different, then the gradient also has units. For example, if the units on the horizontal and vertical axes are seconds and centimetres, respectively, then the units for the gradient are cm/s.

Book B, Unit 6, page 78



Finding the intercepts and gradient of the line y=mx+c

- The gradient is the coefficient m.
- The y-intercept is the constant term c.
- The x-intercept is the solution of mx + c = 0.

Book B, Unit 6, pages 94–5

Book B, Unit 6, page 96

Book B, Unit 6, page 98

When you use this strategy you should also find and plot a third point on the line, as a check.

Book B, Unit 6, pages 99-103

Book B, Unit 7, Example 7, pages 135–136

Book B, Unit 7, page 137

If most of the terms have a minus sign, then you may wish to take this out as well.

If the common factor that you're taking out is the same as one of the terms, then what's left is 1.

Drawing the line y = mx + c (gradient method)

- 1. Mark the point (0, c) that corresponds to the y-intercept.
- 2. Count 1 unit right and m units up from the point, and mark the point that you reach. (If m is negative, then you count down rather than up.)
- 3. Draw the straight line through the two points.

(If the value of m is small, or a fraction, then in step 2 it might be easier to count, say, 2 units right and 2m units up, or 3 units right and 3m units up, and so on – choose a convenient multiple.)

Drawing the line y = mx + c (two-point method)

- 1. Find the coordinates of two points on the line, for example by choosing two values of x and substituting them into the equation to find the corresponding values of y.
- 2. Plot the points and draw the straight line through them.

Finding the equation of a line

- If the line is vertical, then the equation is x = a, where a is the x-coordinate of any point on the line.
- If the line is horizontal, then the equation is y = b, where b is the y-coordinate of any point on the line.
- In all other cases, do the following.
 - 1. If the gradient m is not already known, then calculate it by substituting the coordinates of two points (x_1, y_1) and (x_2, y_2) on the line into

$$m = \frac{y_2 - y_1}{x_2 - x_1}.$$

Then substitute the gradient into the general equation y = mx + c.

- 2. If the y-intercept c is not already known, then substitute the coordinates of a point on the line into the equation of the line from step 1, and solve the resulting equation to find c.
- 3. Use the values of m and c to write down the equation of the line.

Unit 7: Equations and inequalities

Finding the highest common factor of two or more terms

- 1. If the coefficients are integers, write down their highest common factor.
- 2. For each letter appearing in the terms, write down (if possible) the highest power of the letter that is a factor of all the terms.
- 3. Write down anything else that is a factor of all the terms.

Taking out a common factor from an expression

- 1. Find a common factor of the terms (normally the highest common factor).
- 2. Write the common factor in front of a pair of brackets.
- 3. Write what's left of each term inside the brackets.

In complicated cases you may wish to multiply out the brackets again and check that you get the original expression.

Making a variable the subject of an equation

Carry out a sequence of steps. In each step, do one of the following:

- do the same thing to both sides
- simplify one side or both sides
- swap the sides.

Aim to do the following, in order.

- 1. Clear any fractions and multiply out any brackets. To clear fractions, multiply both sides by a suitable expression.
- 2. Add or subtract terms on both sides to get all terms containing the required subject on one side, and all other terms on the other side.
- 3. If more than one term contains the required subject, then take it out as a common factor. This gives an equation of the form

$$\begin{array}{c} \hline \text{expression} \times \hline \end{array} \times \hline \begin{array}{c} \text{required subject} \end{array} = \overline{ \begin{array}{c} \text{expression} \\ \end{array} }$$

4. Divide both sides by the expression that multiplies the required subject.

Solving simultaneous equations: graphical method

- 1. Draw the graph of each equation on the same axes, choosing scales so that the intersection point can be seen.
- 2. The values of the unknowns at the intersection point give the solution.

Determining whether simultaneous equations have a solution

Write the equations in the form

$$y = ax + b,$$

$$y = cx + d.$$

- If the constants a and c are not equal, then the lines representing the equations are not parallel, so the equations have one solution.
- If the constants a and c are equal, then the lines representing the equations are parallel, so the equations do not have a solution. (There is an an exception to this: if the constants b and d are also equal, then the two equations are the same, so there are infinitely many solutions.)

Solving simultaneous equations: substitution method

- 1. Rearrange one of the equations, if necessary, to obtain a formula for one unknown in terms of the other.
- 2. Use this formula to substitute for this unknown in the other equation.
- 3. You now have an equation in one unknown. Solve it to find the value of the unknown.
- 4. Substitute this value into an equation involving both unknowns to find the value of the other unknown.

(Check: Confirm that the two values satisfy the original equations.)

Book B, Unit 7, page 141

Remember that you have to do the same thing to the *whole* of each side, namely one of:

- o add something
- subtract something
- multiply by something
- divide by something non-zero.

Book B, Unit 7, page 145

Book B, Unit 7, page 148

Book B, Unit 7, page 152

Book B, Unit 7, page 155

It may be helpful to divide each equation through by any common factor, and clear any fractions, before you start working with the equations.

Book B, Unit 7, page 165

Book C, Unit 8, page 10

Book C, Unit 8, page 13

Book C, Unit 8, page 14

Book C, Unit 8, page 15

Book C, Unit 8, page 17

Book C, Unit 8, page 19

Book C, Unit 8, page 19

Book C, Unit 8, page 24

Book C, Unit 8, page 24

Solving simultaneous equations: elimination method

- 1. Multiply one or both of the equations by suitable numbers, if necessary, to obtain two equations that can be added or subtracted to eliminate one of the unknowns.
- 2. Add or subtract the equations to eliminate the unknown.
- 3. You now have an equation in one unknown. Solve it to find the value of the unknown.
- 4. Substitute this value into an equation involving both unknowns to find the value of the other unknown.

(Check: Confirm that the two values satisfy the original equations.)

Rearranging an inequality

You can do any of the following things to a correct inequality to obtain another correct inequality.

- Do any of the following to both sides.
 - Add or subtract a number.
 - Multiply or divide by a *positive* number.
 - Multiply or divide by a negative number, if you reverse the inequality sign.
- Simplify one side or both sides.
- Swap the sides, if you reverse the inequality sign.

Unit 8: Geometry

Angles (lines)

- Angles on a straight line add up to 180°.
- Opposite (X) angles are equal.
- Corresponding (F) angles on parallel lines are equal.
- Alternate (Z) angles on parallel lines are equal.

Angles (polygons)

- The interior angles of a triangle add up to 180°.
- The angles in an equilateral triangle are equal (to 60°).
- The base angles in an isosceles triangle are equal.
- Opposite angles in a parallelogram are equal.
- A kite has a pair of opposite equal angles.

Checking whether two triangles are congruent

Check whether the triangles satisfy one of the following conditions.

- The three sides of one triangle are equal to the three sides of the other triangle (SSS).
- Two sides and the included angle of one triangle are equal to two sides and the included angle of the other triangle (SAS).
- Two angles and the included side of one triangle are equal to two angles and the included side of another triangle (ASA).
- Two angles and a side of one triangle in the order angle-angle-side are equal to two angles and a side of the other triangle in the same order (AAS).

Checking whether two triangles are similar

Check whether the triangles satisfy one of the following conditions.

- Two (and hence three) angles of one triangle are equal to two (and hence three) angles of the other triangle.
- The three sides of one triangle are in proportion to the three sides of the other triangle (that is, their ratios are equal).
- An angle of one triangle is equal to an angle of the other triangle, and the sides containing these angles are in proportion (that is, their ratios are equal).

Finding an unknown side of a triangle using a similar triangle

- 1. Show that the triangle is similar to another triangle with known sides.
- 2. Equate the ratios of corresponding sides.
- 3. Solve for (the length of) the unknown side.

Pythagoras' Theorem

For a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides. So, for the right-angled triangle in the margin, $AB^2 = AC^2 + BC^2$.

Converse of Pythagoras' Theorem

If a triangle has sides of lengths a, b and c with $a^2 + b^2 = c^2$, then the angle opposite the side of length c is a right angle.

Areas

- A rectangle with sides a and b has area ab.
- A parallelogram with base b and perpendicular height h has area bh.
- A triangle with base b and perpendicular height h has area $\frac{1}{2}bh$.
- A trapezium with parallel sides a and b, and perpendicular height h, has area $\frac{1}{2}(a+b)h$.

Circles

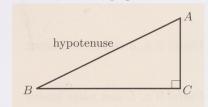
- The circumference of a circle of radius r is $2\pi r$.
- The area of a circle of radius r is πr^2 .

Book C, Unit 8, page 34

Book C, Unit 8, page 42

Book C, Unit 8, page 39

Book C, Unit 8, page 43



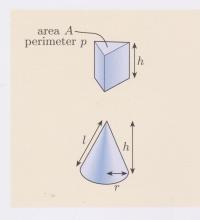
Book C, Unit 8, page 46

Book C, Unit 8, pages 48–51

Book C, Unit 8, page 55

Book C, Unit 8, page 56

Book C, Unit 8, page 61



Book C, Unit 9, page 74

Book C, Unit 9, page 78

Book C, Unit 9, page 73

Book C, Unit 9, page 76

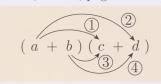
Book C, Unit 9, page 77

Useful if a, d and n are known.

Useful if a, L and n are known.

Useful if a, L and d are known.

Book C, Unit 9, page 82



Book C, Unit 9, page 84

Book C, Unit 9, page 86

Solids

- A *cuboid* of width w, height h and depth d has volume whd and surface area 2wh + 2wd + 2hd.
- A prism (as shown in the margin) with a cross-section of area A and perimeter p, whose height is h, has volume Ah and surface area 2A + hp.
- A cylinder of radius r and height h has volume $\pi r^2 h$ and surface area $2\pi r^2 + 2\pi r h$.
- A cone (as shown in the margin) of radius r, height h and slant height l has volume $\frac{1}{3}\pi r^2 h$ and surface area $\pi r^2 + \pi r l$.
- A sphere of radius r has volume $\frac{4}{3}\pi r^3$ and surface area $4\pi r^2$.

Unit 9: Expanding algebra

Sums of sequences

- The sum of the first n natural numbers is $\frac{1}{2}n(n+1)$.
- The sum of the first n even numbers is n(n+1).
- The sum of the first n odd numbers is n^2 .

The nth term of an arithmetic sequence

The nth term of an arithmetic sequence with first term a and common difference d is given by the formula

$$n$$
th term = $a + (n-1)d$.

The sum of an arithmetic sequence

The sum S of a finite arithmetic sequence with first term a, last term L, common difference d and number of terms n is given by either of the formulas

$$S = \frac{1}{2}n(2a + (n-1)d)$$

or

$$S = \frac{1}{2}n(a+L),$$

where n, if not already known, can be found from the formula

$$n = \frac{L - a}{d} + 1.$$

Multiplying out two brackets

Multiply each term inside the first bracket by each term inside the second bracket, and add the resulting terms. (You can use the acronym FOIL to remember the order: (1) First ac; (2) Outer ad; (3) Inner bc; (4) Last bd.)

Square of a bracket

$$(x+p)^2 = x^2 + 2px + p^2$$
 or $(x-p)^2 = x^2 - 2px + p^2$

Difference of two squares

$$(x-p)(x+p) = x^2 - p^2$$

Factorising $x^2 + bx + c$, where b and c are integers

- 1. Decide whether the factorisation is one of the following special types.
 - The quadratic has no constant term, so it factorises like this: $x^2 + bx = x(x+b)$, for example $x^2 6x = x(x-6)$.
 - The quadratic is a difference of two squares, so it factorises like this: $\frac{1}{2} \left(\frac{1}{2} \frac{1}{2} \right) \left(\frac{1}{2} \frac{1}{2} \frac{1}{2} \right) \left(\frac{1}{2} \frac{1}{2}$
 - $x^{2} p^{2} = (x p)(x + p)$, for example $x^{2} 9 = (x 3)(x + 3)$.
 - The quadratic is a perfect square, so it factorises like this:

$$x^{2} + 2px + p^{2} = (x+p)^{2}$$
, for example $x^{2} - 6x + 9 = (x-3)^{2}$.

2. Otherwise, try to fill in the gaps in the brackets on the right-hand side of the equation

$$x^2 + bx + c = (x,)(x,)$$

with two integers (positive or negative) whose product is c and whose sum is b. You can search systematically for integers with these properties by:

- writing down the factor pairs of c, the constant term
- choosing (if possible) a pair whose sum is b, the coefficient of x.

Factorising $ax^2 + bx + c$, where a, b and c are integers

First method

ullet For each factor pair r, s of a, try to find a factorisation of the form

$$ax^2 + bx + c = (rx \underline{\hspace{1cm}})(sx \underline{\hspace{1cm}}),$$

using a factor pair of c to fill the gaps.

- Multiply out the brackets to check whether the factorisation is successful.
- Keep trying until you obtain a factorisation, or until all factor pairs of c and of a have been exhausted.

Second method

- Find two numbers p, q whose product is ac and whose sum is b.
- Rewrite the quadratic expression by using the two numbers above to split the term in x:

$$ax^2 + bx + c = ax^2 + px + qx + c.$$

• Group the four terms in pairs and take out common factors to give the required factorisation:

$$ax^{2} + bx + c = \underbrace{ax^{2} + px}_{} + qx + c$$

$$= \dots$$

$$= \dots$$

Numbers whose product is zero

If the product of two or more numbers is 0, then at least one of the numbers must be 0.

Book C, Unit 9, page 92

For some quadratic expressions, the methods given on this page will not lead to a factorisation because a quadratic does not necessarily have a factorisation using integers, even if its coefficients are integers.

You have to find only *one* such pair of integers.

Book C, Unit 9, Example 8, pages 97–98

Book C, Unit 9, Example 9, page 98

For the quadratic

$$2x^2 - x - 6$$
,

we need numbers p, q whose product is -12 and whose sum is -1, so we take

$$p = 3, \quad q = -4.$$

Then

$$2x^{2} - x - 6$$

$$= 2x^{2} + 3x - 4x - 6$$

$$= x(2x + 3) - 2(2x + 3)$$

$$= (x - 2)(2x + 3).$$

Book C, Unit 9, page 95

Book C, Unit 9, page 96, 99

For example, the equation

$$2x^2 - x - 6 = 0$$

is equivalent to

$$(x-2)(2x+3) = 0,$$

which gives

$$x - 2 = 0$$
 or $2x + 3 = 0$;

that is.

$$x = 2$$
 or $x = -\frac{3}{2}$.

Book C, Unit 9, page 104

Book C, Unit 9, page 105

Book C, Unit 9, page 107

Book C, Unit 9, page 110

Book D, Unit 14, page 193

Book C, Unit 9, page 116

For example, $a^2 = bc$ becomes $a = (bc)^{1/2}$, that is, $a = \sqrt{bc}$.

Solving a quadratic equation by factorisation

- 1. Rearrange the equation into the form $ax^2 + bx + c = 0$, if it is not already in this form.
- 2. Factorise the LHS.
- 3. Use the fact that if the product of two numbers is zero then at least one of the numbers must be zero.
- 4. Solve the resulting linear equations.

Simplifying an algebraic fraction

Factorise the numerator and denominator (if necessary) and cancel any

common factors. For example, $\frac{2x^2 + 6x}{x^2 - 9} = \frac{2x(x+3)}{(x-3)(x+3)} = \frac{2x}{x-3}$.

Adding or subtracting algebraic fractions

- 1. Make sure that the fractions have a common denominator if necessary, rewrite each fraction as an equivalent fraction.
- 2. Add or subtract the numerators.
- 3. Simplify the answer by cancelling if possible.

Multiplying or dividing algebraic fractions

• To multiply two algebraic fractions, multiply the numerators together and multiply the denominators together:

$$\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$$

• To divide one algebraic fraction by another, multiply the first fraction by the reciprocal of the second fraction:

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}.$$

In each case, you should cancel any common factors that appear.

Clearing algebraic fractions from an equation

Multiply both sides of the equation by a common multiple (often the product) of the denominators of all the fractions that you want to clear.

If the equation is of the form $\frac{A}{B} = \frac{C}{D}$, where A, B, C and D are expressions, then you can cross-multiply to obtain AD = BC.

Both methods are valid only when the values of the variables are such that the denominators of the fractions are non-zero.

Rearranging an equation when the required subject is raised to a power

1. Try to rearrange the equation into the form

the required subject a power = an expression

2. Obtain the required subject on its own on the left-hand side by raising both sides of the equation to the reciprocal of the power.

Unit 10: Quadratics

The graph of the equation $y = ax^2 + bx + c$

If a is positive, then the graph is a u-shaped parabola.

If a is negative, then the graph is an n-shaped parabola.

The graph has the same shape as the graph of $y = ax^2$, but shifted.

The graph crosses the y-axis at (0, c).

The axis of symmetry of the graph is $x = -\frac{b}{2a}$.

Finding the intercepts of the parabola $y = ax^2 + bx + c$

- The x-intercepts are the solutions of the quadratic equation $ax^2 + bx + c = 0$.
- The y-intercept is c.

Finding the vertex of the parabola $y = ax^2 + bx + c$

Use any of the following methods.

- Use the formula x = -b/(2a) to find the x-coordinate of the vertex, then substitute into the equation of the parabola to find the y-coordinate.
- Find the x-intercepts; then the value halfway between them is the x-coordinate of the vertex. Find the y-coordinate by substituting into the equation of the parabola.
- Find the completed square form $y = a(x h)^2 + k$; then the vertex is (h, k).

Sketching the graph of a quadratic function

- 1. Decide whether the parabola is u-shaped or n-shaped.
- 2. Find its intercepts, axis of symmetry and vertex.
- 3. Plot the features found, and hence sketch the parabola.
- 4. Label the parabola with its equation, and make sure that the values of the intercepts and the coordinates of the vertex are indicated.

Solving a quadratic equation graphically

- 1. Obtain a graph of the corresponding quadratic function, using a scale on the x-axis that enables you to read x-coordinates to the desired accuracy of the solution.
- 2. Read off the values of x when y = 0, that is, the x-intercepts.

The quadratic formula

The solutions of the quadratic equation

$$ax^2 + bx + c = 0$$

are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Book C, Unit 10, page 143, 147



Book C, Unit 10, page 144

Book C, Unit 10, page 145, 172

You can check an answer for the vertex of a parabola by plotting the parabola using Graphplotter and reading off the approximate coordinates of the vertex.

Book C, Unit 10, page 146

Book C, Unit 10, pages 148-149

For example, you can use Graphplotter to obtain a graph of a quadratic function.

Book C, Unit 10, page 152

If the coefficients have any common factors, then you may wish to divide the equation through by them before using the formula; and if any of the coefficients are fractions, then you may wish to multiply through by a suitable number to clear them.

Book C, Unit 10, page 157

Book C, Unit 10, page 164

Book C, Unit 10, page 167

Book C, Unit 10, page 171

Book C, Unit 10, Example 9, pages 167–168

$$4x^2 + 8x - 1 = 0$$

$$x^2 + 2x - \frac{1}{4} = 0$$

$$(x+1)^2 - \frac{5}{4} = 0$$

$$(x+1)^2 = \frac{5}{4}$$

$$x+1=\pm \frac{1}{2}\sqrt{5}$$

$$x = -1 \pm \frac{1}{2}\sqrt{5}$$

Book C, Unit 10, page 179

The number of solutions of a quadratic equation

The quadratic equation $ax^2 + bx + c = 0$ has

- two solutions if $b^2 4ac > 0$ (the discriminant is positive)
- one solution if $b^2 4ac = 0$ (the discriminant is zero)
- no solutions if $b^2 4ac < 0$ (the discriminant is negative).

Completing the square in a quadratic expression

• If the quadratic is of the form $x^2 + bx$, write

$$x^2 + bx = (x+p)^2 - p^2$$
 (where p has the value $b/2$)

and evaluate the constant term $-p^2$.

• If the quadratic is of the form $x^2 + bx + c$, write

$$x^2 + bx + c = (x+p)^2 - p^2 + c$$
 (where p has the value $b/2$)

and collect the constant terms.

• If the quadratic is of the form $ax^2 + bx + c$, write

$$ax^2 + bx + c = a(x^2 + qx) + c$$
 (where q has the value b/a)
= $a((x+p)^2 - p^2) + c$ (where p has the value $q/2$).

Then multiply out the *outer* brackets, and collect the constant terms.

Solving a quadratic equation by completing the square

The working for $4x^2 + 8x - 1 = 0$ is in the margin.

- 1. Divide through by the coefficient of x^2 .
- 2. Complete the square.
- 3. Get the constant term on the RHS.
- 4. Take the square root of both sides.
- 5. Get x by itself on the LHS.

Solving a maximisation (or minimisation) problem

- 1. Identify the quantity to be maximised (or minimised) and the quantity that it depends on, and denote each quantity by a variable.
- 2. Find a formula for the variable to be maximised (or minimised) in terms of the variable that it depends on.
- 3. If this gives a quadratic function, then find the vertex of its graph.

The required maximum (or minimum) value is equal to the second coordinate of the vertex. This value is achieved when the independent variable is equal to the first coordinate of the vertex.

Unit 11: Statistical pictures

Characteristics of boxplots

- A boxplot is composed of four sections (two whiskers at either end and two sections within the central box), each of which contains approximately the same number of data values.
- Where a particular boxplot section is narrow, this indicates a dense concentration of the data, whereas a wide section indicates where the data are more sparsely spread.

Book D, Unit 11, page 12, 16

Book D, Unit 11, page 33

Uniform random numbers

- When a fairly small run of uniform random numbers is chosen, the degree of disorderliness in the numbers is often surprisingly high.
- With larger runs, the frequencies tend to settle down and become approximately equal.
- Knowing the extent of random fluctuations for a given sample size provides a benchmark against which to interpret experimental data.

Unit 12: Trigonometry

Trigonometric ratios

In a right-angled triangle with an acute angle θ , the sine, cosine and tangent of θ are given by

$$\sin\theta = \frac{\mathrm{opp}}{\mathrm{hyp}}, \quad \cos\theta = \frac{\mathrm{adj}}{\mathrm{hyp}}, \quad \tan\theta = \frac{\mathrm{opp}}{\mathrm{adj}}, \qquad \begin{array}{l} \text{(mnemonic:} \\ \text{SOH CAH TOA)} \end{array}$$

where hyp, opp and adj are the lengths of the hypotenuse, the side opposite θ and the side adjacent to θ , respectively.

Trigonometric identities

$$\cos \theta = \sin(90^{\circ} - \theta)$$
 or $\cos \theta = \sin(\frac{1}{2}\pi - \theta)$
 $\sin \theta = \cos(90^{\circ} - \theta)$ or $\sin \theta = \cos(\frac{1}{2}\pi - \theta)$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$
, provided that $\cos \theta \neq 0$

$$\sin^2\theta + \cos^2\theta = 1$$

$$\cos(-\theta) = \cos \theta, \quad \sin(-\theta) = -\sin \theta, \quad \tan(-\theta) = -\tan \theta$$

Sine Rule

In a triangle with sides of length a, b, c and opposite angles A, B, C, respectively:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
 or $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$.

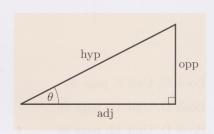
Cosine Rule

In a triangle with sides of length a, b, c and opposite angles A, B, C, respectively:

$$a^{2} = b^{2} + c^{2} - 2bc \cos A,$$

 $b^{2} = c^{2} + a^{2} - 2ca \cos B,$
 $c^{2} = a^{2} + b^{2} - 2ab \cos C.$

Book D, Unit 12, page 61



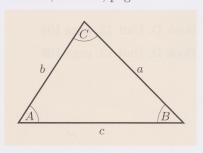
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Book D, Unit 12, page 74, 91

Book D, Unit 12, page 74

Book D, Unit 12, page 96

Book D, Unit 12, page 77

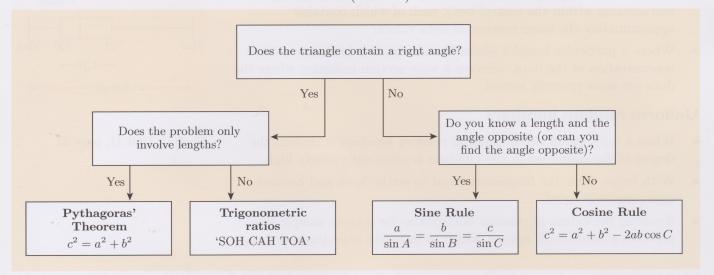


Book D, Unit 12, page 80

Book D, Unit 12, page 75, 83

Solving a triangle

- 1. Sketch a diagram showing the known measurements.
- 2. Using the following decision tree, write down an equation relating the unknown to (some of) the known measurements.



3. Solve the equation to find the unknown. If the unknown is an angle then it may be necessary to apply \sin^{-1} , \cos^{-1} or \tan^{-1} , and in the case of \sin^{-1} you should consider whether the required angle is the obtuse angle $180^{\circ} - \theta$ rather than the acute angle θ returned by your calculator.

Finding the area of a triangle

- If the base b and perpendicular height h are known, calculate $\frac{1}{2}bh$.
- If two sides a, b and the included angle θ are known, calculate $\frac{1}{2}ab\sin\theta$.
- If the three sides a, b and c are known, use the semi-perimeter $s = \frac{1}{2}(a+b+c)$ to calculate $\sqrt{s(s-a)(s-b)(s-c)}$ (Heron's formula).

Book C, Unit 8, page 50 Book D, Unit 12, page 86 Book D, Unit 12, page 86

Book D, Unit 12, page 103

Converting between degrees and radians

 $360^{\circ} = 2\pi$ radians, so

- angle in radians = $\frac{\pi}{180}$ × angle in degrees,
- angle in degrees $=\frac{180}{\pi} \times \text{angle in radians}.$

Finding the area of a sector or the length of an arc

Determine the radius r of the circle with which the arc or sector is associated. Then

- arc length = $r\theta$,
- area of sector = $\frac{1}{2}r^2\theta$,

where θ (measured in radians) is the angle subtended by the arc or the angle of the sector.

Book D, Unit 12, page 104 Book D, Unit 12, page 106

Special angles table

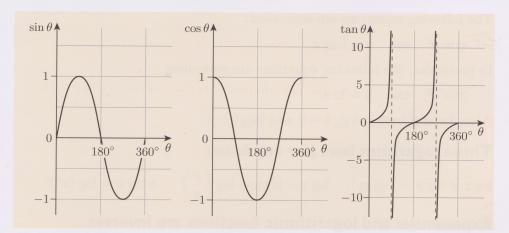
θ in degrees	θ in radians	$\sin \theta$	$\cos \theta$	$\tan \theta$
0°	0	0	1	0
30°	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
45°	$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
60°	$\frac{\pi}{3}$	$\frac{\sqrt{2}}{\sqrt{3}}$	$\frac{1}{2}$	$\sqrt{3}$
90°	$\frac{\pi}{2}$	1	0	· —

Book D, Unit 12, Subsection 1.4 and page 108

Book D, Unit 12, pages 94–95

Trigonometric graphs

The graphs of the sine, cosine and tangent functions are shown below.



Unit 13: Exponentials

Discrete exponential change

Suppose that a positive quantity changes in steps, where its value at each step is obtained by multiplying its value at the previous step by the same scale factor b. If the starting number is a, then the value after n steps is ab^n . If b > 1 then the quantity grows; if 0 < b < 1 then it decays.

Scale factors for percentage increases and decreases

To increase a number by r%, multiply it by the scale factor $\frac{100+r}{100}$

To decrease a number by r%, multiply it by the scale factor $\frac{100-r}{100}$

Discrete exponential change over different numbers of steps

Suppose that a quantity changes by the scale factor b at each step. Then every i steps, it changes by the scale factor b^i .

Continuous exponential change over different periods of time

Suppose that a quantity is subject to continuous exponential change by the scale factor b during each unit of time.

Then over i units of time, it changes by the scale factor b^i .

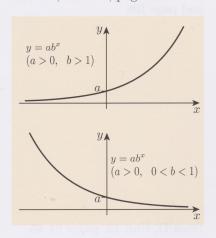
Book D, Unit 13, pages 119-120

Book D, Unit 13, page 121

Book D, Unit 13, page 134

Book D, Unit 13, page 137

Book D, Unit 13, page 145



Book D, Unit 13, pages 154-156

Book D, Unit 13, page 161 Here n can be any number, but x and y must be positive since only positive numbers have logarithms.

Book D, Unit 13, page 165

Book D, Unit 13, Examples 10 and 11, pages 162–163

It is usually best to use either logarithms to base 10 or natural logarithms, as these are easily available from your calculator. If p = e, use natural logarithms and simplify or evaluate the resulting expression for x.

Book D, Unit 13, page 168

Graphs of equations of the form $y = ab^x$

• If a > 0 then the graph lies entirely above the x-axis.

If also

b>1, then the graph is an exponential growth curve;

0 < b < 1, then the graph is an exponential decay curve;

b = 1, then the graph is a horizontal line.

- If a < 0 then the graph lies entirely below the x-axis and is neither an exponential growth curve nor an exponential decay curve.
- The x-axis is an asymptote (except when a = 0 or b = 1).
- The y-intercept is a.
- The closer the value of b is to 1 (and the closer the value of a is to 0) the flatter is the graph.

Logarithms

The following equations are equivalent.

$$x = b^y$$
 and $y = \log_b x$.

In particular, the following equations are equivalent.

$$x = e^y$$
 and $y = \ln x$.

Also, for any base b, $\log_b 1 = 0$ and $\log_b b = 1$.

Three logarithm laws (for any base)

$$\log x + \log y = \log(xy), \quad \log x - \log y = \log\left(\frac{x}{y}\right), \quad n \log x = \log(x^n)$$

Exponential and logarithmic functions are inverses

$$\ln\left(e^x\right) = x, \quad e^{\ln x} = x$$

More generally,

$$\log_b(b^x) = x, \quad b^{\log_b x} = x$$

Solving an exponential equation

1. Rearrange the equation so that it has the form

$$p = q,$$

where x is the unknown and p and q are numbers.

2. Take logarithms of both sides and use the third logarithm law (above) to write the equation in the form

(an expression involving
$$x \times \log p = \log q$$
).

3. Solve the resulting equation for x.

Finding the doubling or halving time of a quantity

Calculate (if not already known) the scale factor b by which the quantity changes during one unit of time.

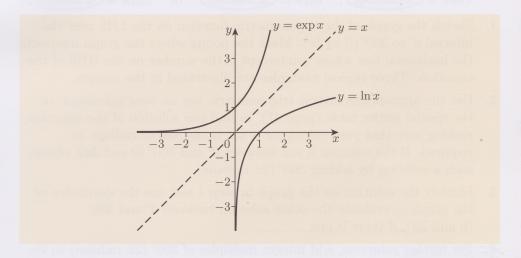
- If the quantity grows (that is, if b > 1), then solve $b^i = 2$.
- If the quantity decays (that is, if 0 < b < 1), then solve $b^i = \frac{1}{2}$.

The solution i is the required doubling or halving time, in the same units of time.

Graphs of inverse functions

Book D, Unit 13, page 170

The graphs of a pair of inverse functions are reflections of each other in the line y=x.



Unit 14: Mathematics everywhere

Estimating the probability of an event

- 1. Consider a large number of cases that *could* result in the event that you are interested in.
- 2. Count the number of cases that do result in the event.
- 3. Divide the number of cases that result in the event by the total number of cases. The quotient provides an estimate of the probability.

Solving trigonometric equations (aided by CAST)

To solve an equation of the form

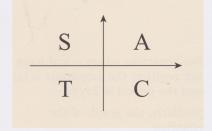
$$\cos \theta =$$
a number $, \sin \theta =$ a number $\cos \theta =$ a number $,$

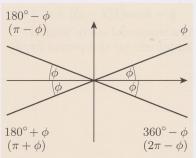
where the number on the RHS is not zero.

- 1. Use the CAST diagram to find the quadrants of the solutions.
- 2. Use your calculator, or the special angles table (page 47), to find an angle in the first quadrant whose cosine, sine or tangent (as appropriate) is the magnitude of the number on the RHS. Make sure that your calculator is set to degrees or radians as required.
- 3. Use the results of steps 1 and 2 and the related angles diagram (expressed in radians if required) to find the solutions in the interval 0° to 360° (0 to 2π).
- 4. For further solutions, add integer multiples of 360° (2π radians) to the solutions mentioned in step 3.

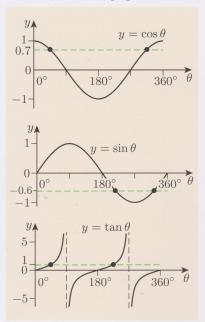
Book D, Unit 14, page 181

Book D, Unit 14, pages 199–200

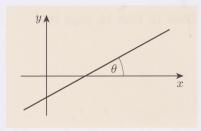




Book D, Unit 14, pages 206-207



Book D, Unit 14, page 202



Book D, Unit 14, pages 225–226

For equations where a and b are not positive, the amplitude is |a| and the period is $2\pi/|b|$.

Similarly, the graph of the equation

$$y = a\cos(b(x-c)) + d$$
 is a sinusoidal curve with amplitude $|a|$ and period $2\pi/|b|$.

Solving trigonometric equations (aided by sketch graphs)

To solve an equation of the form

$$\cos \theta =$$
 a number , $\sin \theta =$ a number or $\tan \theta =$ a number .

- 1. Sketch the graph of the trigonometric function on the LHS over the interval 0° to 360° (0 to 2π). Mark the points where the graph intersects the horizontal line whose *y*-intercept is the number on the RHS of the equation. Three typical examples are illustrated in the margin.
- 2. Use the appropriate inverse trigonometric key on your calculator, or the special angles table (page 47), to find one solution of the equation, making sure that your calculator is set to degrees or radians as required. If the solution is not between 0° and 360° (0 and 2π), obtain such a solution by adding 360° (2π radians).
- 3. Identify the solution on the graph in step 1 and use the symmetry of the graph to evaluate the other solution between 0° and 360° (0 and 2π), if there is one.
- 4. For further solutions, add integer multiples of 360° (2π radians) to the solutions mentioned in step 3.

Gradient and angle of inclination of a straight line

For any straight line with angle of inclination θ ,

gradient =
$$\tan \theta$$
.

(The angle of inclination is measured when the line is drawn on axes with equal scales.)

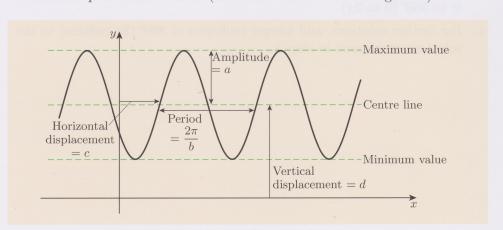
The graph of a general sine function

The graph of the equation

$$y = a\sin(b(x-c)) + d,$$

where a and b are positive, and c and d can take any value, has the following features.

- a is the **amplitude**: the distance between the centre line and the maximum (or minimum) value.
- b tells you the period, which is equal to $2\pi/b$.
- c is the **horizontal displacement**: the amount that the graph of $y = a\sin(bx) + d$ is shifted to the right to obtain the graph of $y = a\sin(b(x-c)) + d$. (The shift is to the left if c is negative.)
- d is the vertical displacement: the amount that the centre line is shifted up from the x-axis. (The shift is down if d is negative.)



5 SI units

The Système Internationale d'Unités (SI units) is an internationally agreed set of units and symbols for measuring physical quantities.

Some of these are base units, such as:

```
metre symbol m (measurement of length)
second symbol s (measurement of time)
kilogram symbol kg (measurement of mass)
kelvin symbol K (measurement of temperature).
```

There are also derived units, which are used for quantities whose measurement combines base units in some way. Some of these are

```
area m^2 (metres squared or square metres) volume m^3 (metres cubed or cubic metres) speed m/s (metres per second) acceleration m/s^2 (metres per second per second).
```

The metric unit litre (l) is equivalent to $0.001 \,\mathrm{m}^3$ (or $1000 \,\mathrm{cm}^3$).

Prefixes may be added to units. Commonly used prefixes are:

```
10^{-9}
                              (e.g. nanogram, ng)
n
       nano
                      10^{-6}
                              (e.g. microsecond, \mus)
       micro
\mu
                     10^{-3}
       milli
                              (e.g. millisecond, ms)
m
                     10^{-2}
                              (e.g. centimetre, cm)
c
       centi
                or
                     10^{3}
k
       kilo
                or
                              (e.g. kilogram, kg)
M
       mega
                     10^{6}
                              (e.g. megagram, Mg).
```

6 The Greek alphabet

A	α	alpha	N	ν	nu
В	β	beta	Ξ	ξ	xi
Γ	γ	gamma	O	0	omicron
Δ	δ	delta	П	π	pi
E	ε	epsilon	P	ρ	rho
Z	ζ	zeta	\sum	σ	sigma
Н	η	eta	T	au	tau
Θ	θ	theta	Y	v	upsilon
Ι	ι	iota	Φ	ϕ	phi
K	κ	kappa	X	χ	chi
Λ	λ	lambda	Ψ	ψ	psi
M	μ	mu	Ω	ω	omega



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